A 60-HOUR COURSE

IN

Kinematic Drawing

BY

HARVEY D. WILLIAMS.

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Introduction.

1. Kinematics is the science of time and space or pure motion. It regards material bodies as having only the two attributes of position and form. A change of position is a displacement, and a change of form is a strain. Thus there are two kinds of motion, but it is only with displacement that we are at present concerned.

2. A free body, or a body capable of any displacement, has six degrees of freedom — three translations in directions which are neither parallel nor in the same plane, and three rotations about axes which are neither parallel nor in the same plane. Such a body can take any position in any part of space.

3. Corresponding to the six degrees of freedom are six degrees of constraint. For each degree of constraint that is applied to a body one degree of freedom is lost. Thus a body may have five degrees of freedom and one of constraint, or four of freedom and two of constraint, etc., as illustrated in Fig. 1.

4. A body has one degree of freedom and five of constraint when each point in it that moves is free to describe one line and only one. From our point of view this case is much more important than the others, and the term constrained motion will be restricted to motion with one degree of freedom. Mechanism may be defined as the science of constrained motion; i. e., motion of one degree of freedom.

5. All motion about which we can know anything is relative. Hence

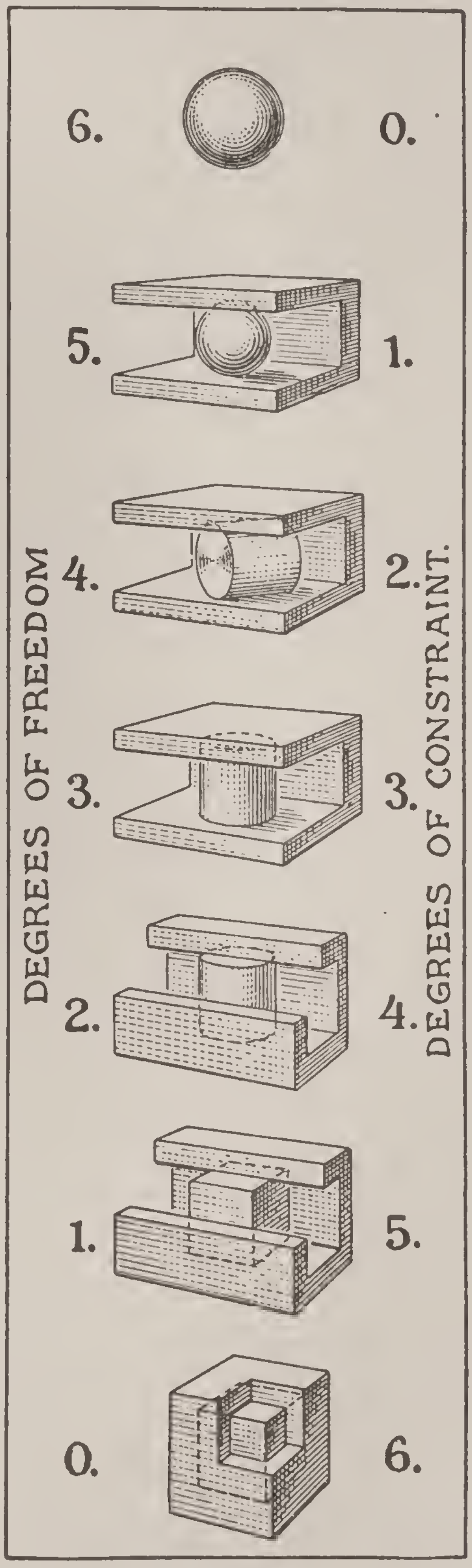


Fig. 1.

there will be under consideration in any given case at least two bodies. When these two bodies move in contact with each other they constitute a kinematic pair. Each of the bodies is a kinematic link, and the surfaces which determine their relative motion are called elements.

6. A single pair of elements may allow more than one degree of freedom, but it is always possible to connect it with other pairs in such a way as to produce perfect constraint and so make it available for use in mechanism.

7. When the elements of a pair are of equal and constant curvature there may be surface contact between them during motion, otherwise there can be only line or point contact. Pairs are thus divided into two classes: primary or lower, those in which surface contact is possible; and higher or secondary, those in which surface contact is impossible.

8. The variety in the forms of elements of higher pairs is without limit; since any two surfaces may touch at a point, and any two surfaces generated by the motion of the same line may have line contact.

9. The number of surfaces of constant curvature is, however, limited to six, of which three have constant curvature in one direction only and three have constant curvature in more than one direction. Hence there will be six primary pairs, as in Fig. 2.

10. One of the elements of a primary pair may be reduced to a number of lines or points without affecting the motion provided the other element is left intact. Thus three points, or a right line and a point, may be equivalent to a plane, and four points, or a circle and a point, may be equivalent to a sphere, etc. A round shaft will have the same motion whether fitted to a round or a square hole, but in the latter case there will be four lines of contact instead of a cylindrical surface. Primary pairs are frequently given line or point contact in this way for the sake of accuracy, as, for example, the *V* supports of a level or transit instrument. Chances of inaccuracy due to the presence of dust evidently decrease with the area of the contact surface. The avoidance of friction is another reason for making similar changes, as in knife-edge fulcrums and roller or ball bearings.

11. By combining the primary pairs it is easy to get very complex motions. But these motions, however complex, are always capable of being expressed by mathematical equations. It follows,

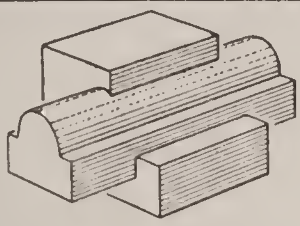
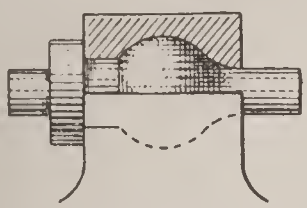
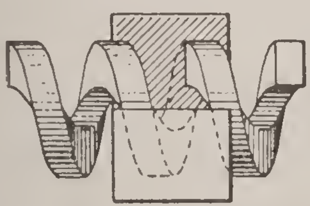
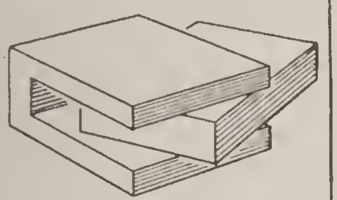
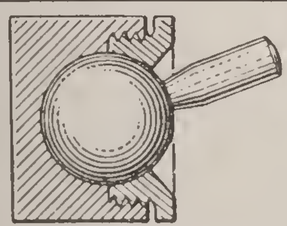
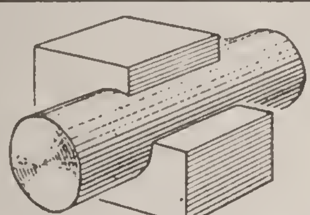
<table>
<tr><th>EXAMPLE</th><th>SURFACE</th><th colspan="2">NAME</th><th>MOTION</th></tr>
<tr><td></td><td>ANY PRISM OR CYLINDER EXCEPT A RIGHT CIRCULAR CYLINDER.</td><td colspan="2">SLIDING PAIR</td><td>TRANSLATION IN ONE DIRECTION</td></tr>
<tr><td></td><td>ANY SURFACE OF REVOLUTION EXCEPT A SPHERE OR A RIGHT CIRCULAR CYLINDER.</td><td colspan="2">TURNING PAIR</td><td>ROTATION ABOUT A FIXED AXIS</td></tr>
<tr><td></td><td>ANY HELICAL SURFACE OF UNIFORM PITCH</td><td colspan="2">TWISTING PAIR</td><td>ROTATION ABOUT AN AXIS AND TRANSLATION IN DIRECTION OF AXIS IN FIXED RATIO TO THE ROTATION.</td></tr>
<tr><td></td><td>PLANE</td><td rowspan="3">SURFACES OF CONSTANT CURVATURE IN MORE THAN ONE DIRECTION</td><td rowspan="3">PARTIALLY CONSTRAINED PRIMARY PAIRS.</td><td>TWO TRANSLATIONS AND ONE ROTATION ALL IN ONE PLANE.</td></tr>
<tr><td></td><td>SPHERE</td><td>THREE ROTATIONS ABOUT AXES WHICH HAVE ONE POINT IN COMMON.</td></tr>
<tr><td></td><td>RIGHT CIRCULAR CYLINDER</td><td>TRANSLATION ROTATION OR TWIST.</td></tr>
</table>

Fig. 2.

therefore, that no such combination can be made to give a purely arbitrary motion.

12. Primary pairs have several structural advantages, which make them very important to the designer. Their forms are simple and easily produced by machinery. Their wear is slight, owing to the surface contact, and when worn they are easily adjusted without changing the character of the motion. It is customary to make the material of one element much harder than the other, in order to preserve the original form of the surface, for if both surfaces are allowed to wear they may gradually lose their surface contact or

become transformed into another primary pair. Thus the sliding pair may become a turning pair. This has been known to occur with an engine slide valve, the originally flat valve seat of which became transformed into a surface of revolution of less than 24″ radius.

13. By means of a higher pair any motion whatever may be produced. One element can always be a surface of constant curvature, and if the motion cannot be produced by primary pairs at least one element must be a surface of varying curvature. In a perfectly constrained higher pair there will be more than one contact, and each element will consist of two or more surfaces, which are related to each other in such a way that when certain ones are known the others may be deduced from them. Thus the acting faces of gear teeth are not wholly arbitrary, but half of them are deducible from the other half.

14. An element of varying curvature will generally wear unequally in its different parts, and the slightest wear, even when uniformly distributed, will change the character of the motion. Matters are still worse when the element consists of two or more irregular surfaces having a proper mathematical relation to each other. Hence, it is that when higher pairs are used in machinery they are arranged so as to give only one or two degrees of constraint, and other constraint is obtained by connecting them with primary pairs. (§ 6.)

15. Any plane or spheric motion can be produced by a pure rolling of two surfaces on each other. Thus the wear may be almost eliminated in many cases by substituting rolling for sliding contact.

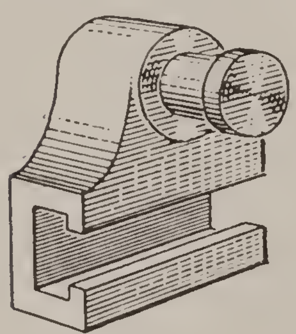


Fig. 4.

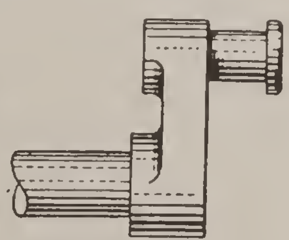


Fig. 3.

16. In a machine each group of pieces having one or more elements, in the latter case without relative motion, is a link. Fig. 3 shows a link having elements of two turning pairs. Fig. 4, a link having an element of a turning pair and an element of a sliding pair. Links may be very simple in form, as in Fig. 5, which con-

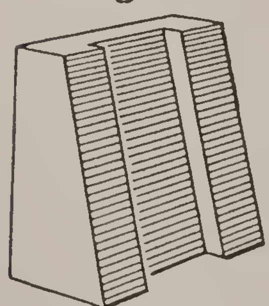


Fig. 6.

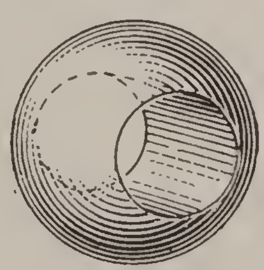


Fig. 5.

nects an element of a partially constrained turning pair with one of a partially constrained twisting pair, or Fig. 6, which connects elements of two sliding pairs, one partially and the other wholly constrained. A compound link is one containing more than two elements, as in Fig. 7.

17. A number of links connected together by their paired elements constitute a kinematic chain. (Fig. 8.) A kinematic chain in which the motion of each link in relation to every other link is constrained is a closed chain or linkage. (Fig. 9.)

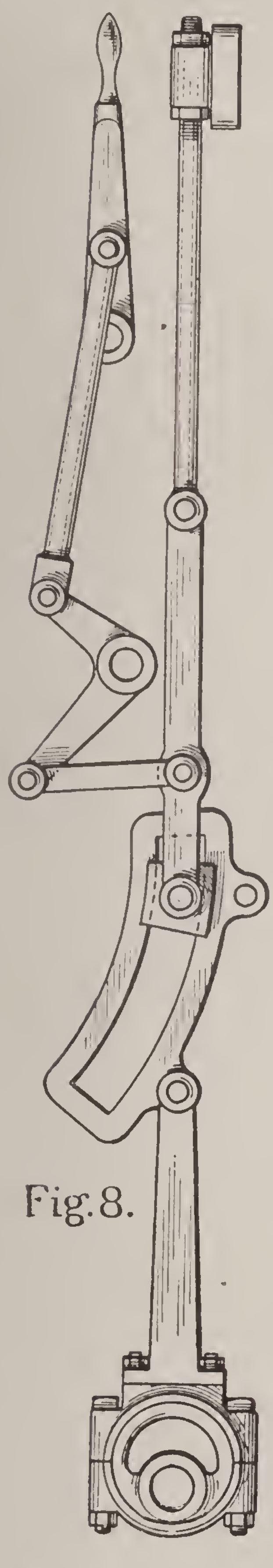


Fig. 8.

18. In designing machinery conditions are imposed by kinematics wherever there is constrained motion. Frequently, one very simple condition is all that it imposes; for example, the rubbing surface of an eccentric strap must be a surface of revolution, since it is an element of a turning pair. In such cases the designer is concerned principally with questions of strength, durability, facility of repair, cost of construction, etc. Such considerations are nearly always leading ones, and in comparatively few cases do the kinematic requirements become so important as to need special investigation by the designer.

Fig. 7.

19. When kinematic requirements do present themselves, however, they are imperative, and there is no such thing as a factor of safety in dealing with them.

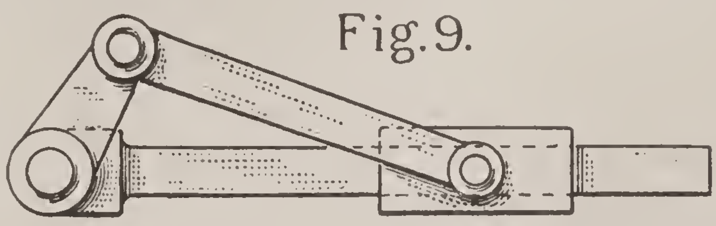


Fig. 9.

20. Care must be taken to distinguish between requirements that are essentially kinematic and those that result from mere expediency. A gear tooth may, within certain limits, be made of any length. There are good reasons for making it long and equally good reasons for making it short, and the ground on which the length is finally decided may be the farthest removed from

kinematics, for it is simply given a length such that it is easily measured and easily described to the end that all who have anything to do with the dimension may avoid mistakes. With the curved face of the tooth, however, the case is different, for here a single requirement is imperative, and the shape is defined with mathematical exactness, and must be produced with all attainable accuracy.

DRAWING INSTRUMENTS.

21. Accuracy is of the first importance. Hence due care must be taken in the selection and use of drawing instruments. The drawing board and T square are apt to warp, and a few moments spent, now and then, in putting them in proper condition will save much annoyance. The face of the drawing board and its left-hand edge must be plane surfaces at right angles to each other. The head of the T square must have a plane surface at right angles to the blade, and the edge of the blade must be straight. When not in use the T square should be suspended vertically or lie, blade down, on a flat surface. Rubber triangles will warp if left in the sun.

22. Use a hard pencil and sharpen it to a conical point. The chisel point is well enough where nothing but straight lines are to be drawn, but it is objectionable for drawing irregular curves, and utterly useless for laying off dimensions from a scale. The conical point must, of course, be kept sharp.

23. One double-jointed compass with removable points is worth more than a whole assortment of compasses with rigid legs. This is very important, and fault is sure to be found with work in which the latter instruments are used. In general it may be said that a few carefully selected instruments, kept in good order, are much better than a large collection of the cheaper variety. The needle and divider points should be sharp enough so that the mere weight of the instrument is sufficient to hold them in place. The compass lead should be sharpened to a chisel point, care being taken that the edge is parallel to the axis of the compass joint.

24. In using the compasses the legs should be bent so as to stand normal to the paper, and in general the instruments should be held in as nearly vertical planes as possible.

25. When a line is to be drawn through two points place the pencil point at one of them and work the straight edge against it as a centre until it touches the other point.

26. Right lines and circles can be bisected, trisected, etc., much more readily by trial than by geometrical constructions if the error of each trial be noted and the dividers opened or closed by the estimated fractional amount of the error. To divide into six parts, first bisect and then trisect each half, and similarly for any other number of divisions when the number can be resolved into factors.

27. When a point has been located indicate its position by two cross lines or by a minute circle around the point. Punching a hole through the paper or obscuring the point with a black smudge of a soft pencil is no indication of its position.

28. The solution of problems involving the relative motion of two bodies can often be facilitated by the use of tracing paper. For this purpose a paper should be selected which will not warp from the moisture of the hands. Two or three fine sewing needles inserted in wooden or sealing-wax handles will be found convenient for laying off dimensions from scales, and also for use as instantaneous centers in connection with the tracing paper.

IRREGULAR CURVES.

29. Irregular curves, or curves other than the circle, will occur in nearly all of the following problems. An irregular curve may be determined by a series of points through which the curve passes, in which case the curve is said to be the locus of the point, or it may be determined by a series of lines, either straight or curved, to each of which the curve is tangent, in which case the curve is said to be the envelope of the lines.

30. Curves determined in either of these ways are drawn by sweeps, the use of which requires special care. First, draw the curve free hand lightly with a soft pencil, and as accurately as possible, in order that the eye may catch the general form. Then for each part of the curve select a portion of a sweep that has about the same curvature, noting at the same time the rate at which the curvature changes. By turning the sweep over, the curvature may be made to increase in either direction. Apply the edge of the sweep to the curve and by trial find a position where the coincidence is the closest, then, with the sweep to guide the pencil, draw that portion of the curve. The difficulty is in selecting the sweeps, and it is evidently of advantage to have a large variety from which to choose.

31. There is an adjustable curved ruler, consisting of a steel tape and a lead bar, by means of which irregular curves can be easily drawn. The ruler is simply bent to the required shape, where it remains by virtue of the ductility of the lead. It does very well where the curvature is only moderate.

32. Let any curve A be cut from a piece of cardboard and suppose an inelastic cord stretched along the curved edge. If the cord be unwound, holding it taut, a point in it will describe a curve B, which is the involute of A, and A is the evolute of B. Since any point in the cord may describe the involute, it follows that for each evolute there are an infinite number of involutes. Since the straight part of the cord is at each instant the radius of curvature of the involute, and therefore normal to it, it follows that involutes of the same evolute are parallel curves.

33. It sometimes happens that it is just as easy to determine the evolute of a required curve as the curve itself. When this is true there is afforded a simple method of drawing the required curve. For if a series of points be found in the evolute, each one may be made the centre of a circular arc, which will agree very closely with a portion of the required curve. It only remains to see that adjacent circular arcs always join at a common normal.

GRAPHICAL CONSTRUCTIONS.

34. In order that the graphical constructions may be easily followed the lines will, in many cases, be numbered 1, 2, 3, 4, etc., in the order in which they are drawn, letters being used to designate points of intersection, subdivision, etc. The following constructions are of a general character, and are applicable to curves whose mathematical properties are unknown. Where exact methods are known they should be given the preference.

35. In solving problems in connection with gear teeth, it will frequently be necessary to lay off equal spaces along the arcs of different curves. On circles having the same radius, equal arcs can be spaced off with a pair of dividers, but when the radii are unequal an empirical method based on the principle illustrated in Fig. 10 is used. If $O\ B$ is one fourth of $O\ A$ a circular arc through A and with B as a centre will cut equal arcs from the several circles, so that arc $O\ C =$ arc $O\ D$

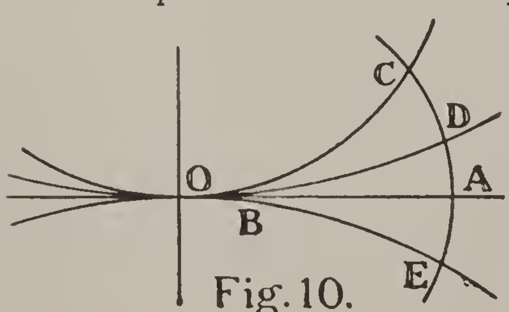


Fig. 10.

$= O\ A =$ arc $O\ E$. The circles are tangent to each other and to the straight line at O. Arcs of equal length found in this way may be sub-divided by continued bisecting (§ 26), thus giving the series of spaces required. This method should not be applied to arcs of over 60°, as, being empirical, it is not absolutely accurate.

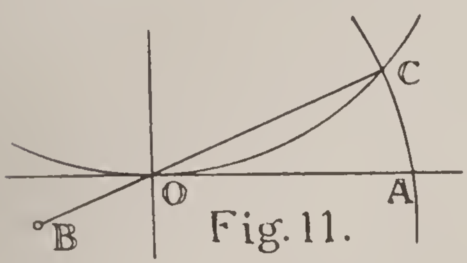


Fig. 11.

36. The construction shown in Fig. 11, deduced from the preceding, gives a means of developing a circular arc into a straight line. $O\ C$ is the given arc. Prolong its chord beyond O, making $O\ B$ equal to one half of the chord $O\ C$. A circle through C, center at B, cuts from the tangent at O a distance $O\ A$, equal to the arc $O\ C$.

37. To find a tangent point on an irregular curve: Let the line 1 (Fig. 12) be tangent to the curve 2. To find the point of tangency, draw two or three chords (as 3, 4) parallel to the tangent, and through the extremities of each chord draw parallel lines 5, 6, 7, 8, and on these lines measure off, from where each intersects the tangent, distances equal to the respective chords; thus, $C\ D = A\ B = E\ F$. A smooth curve (9) passing through the points so found will cut the given curve at the required tangent point.

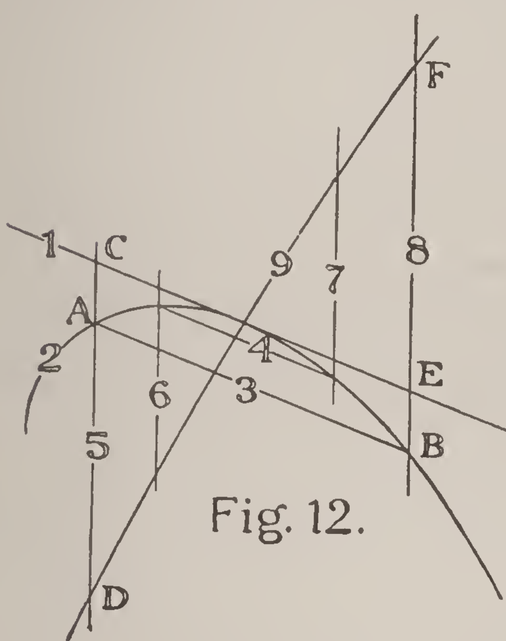


Fig. 12.

38. To draw a tangent through a given point on an irregular curve: Let 1 (Fig. 13) be the given curve and A the point through which a tangent is to be drawn. Draw a circular arc (2) with center at A. On a secant (3) passing through A lay off $C\ D = A\ B$. The locus of D will be a curve (4) which cuts the arc 2 at E, a point in the required tangent.

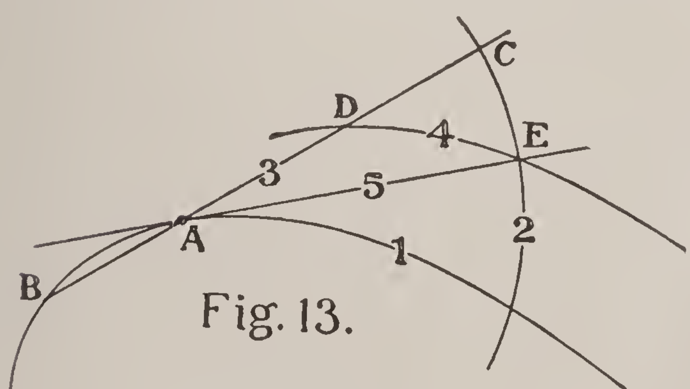


Fig. 13.

39. To draw the evolute of an irregular curve: Let 1 (Fig. 14) be the given curve, 2 a tangent and A its tangent point. (§§ 37, 38.) Draw the normal 3. Find by trial the center (B) of a circle that agrees with a reasonably small portion of the

given curve near A, say from A to C. Draw the normal $C\ B$ and find by trial the center (D) of an arc ($C\ E$) that agrees with another portion of the curve, and so on to the end of the curve. The broken line, $B\ D\ F\ H$, is the evolute required. (§ 33.)

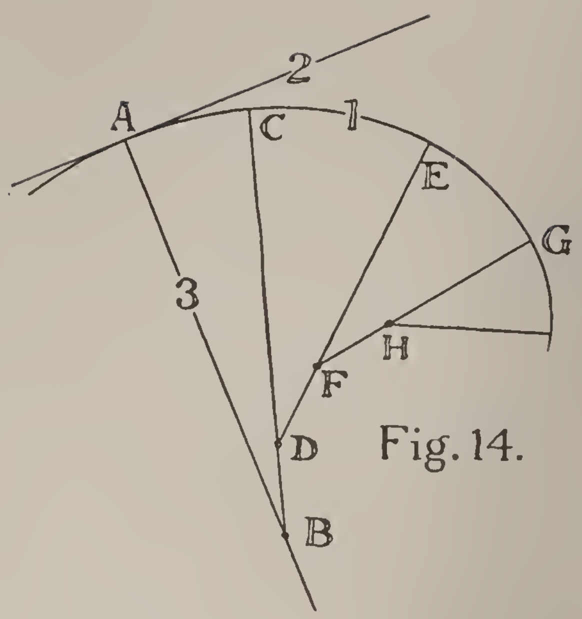


Fig. 14.

40. To lay off equal spaces along the arc of an irregular curve: Draw a fine straight line (Fig. 15) on a strip of tracing paper (§ 28) and divide it into parts at $A\ B\ C\ D$, etc., equal to those required on the curve. Lay off one quarter of a part at Aa, Bb, Cc, etc. Place the tracing over the given curve so that the point A is on the curve. Insert a needle (§ 28) at A and turn the tracing about it as a center until the straight line is tangent to the curve at A. Insert another needle at a, and, removing the first needle, turn the tracing until point B is on the curve. Insert a needle at B, remove the one at a and turn again until B is the tangent point. Then place a needle at b and remove the one at B and rotate until C is on the curve, etc. This method is based on the one described in § 35, and is subject to the same limitations.

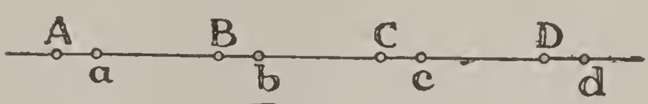


Fig. 15.

41. To lay off equal spaces along circular arcs or along irregular curves when their evolutes are known: Draw on tracing paper three lines as in Fig. 16, making $A\ B$ equal to one of the required spaces. $C\ A$ and $B\ D$ are at right angles to $A\ B$, and each is longer than the longest radius of curvature of the given curve. Lay off $A\ a = B\ b =$ one-fourth of $A\ B$. Place the tracing over the given curve so that the point A is on the curve. Insert a needle at A and turn the tracing about it as a center until $A\ C$ is tangent to the evolute, or, in case of a circle, until $A\ C$ passes through its center. Insert a needle at a, remove the one at A, and turn the tracing until B is on the curve. Insert a needle at B, remove the one at a and turn until $B\ D$ touches the evolute.

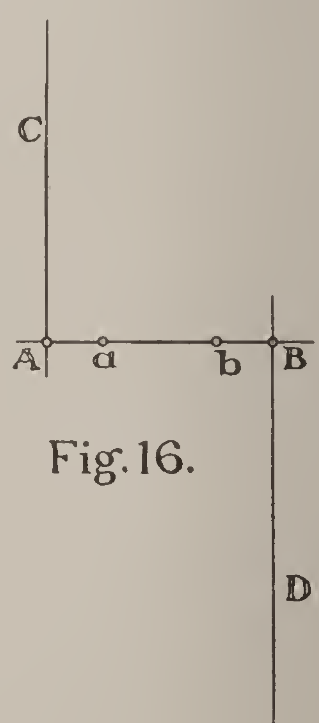


Fig. 16.

42. To draw a parallel to a given curve: Let 1 (Fig. 17) be the given curve. With center on 1 and radius equal to the distance between the curves draw a circular arc, 2. The required curve will be the envelope of the arc 2. By drawing a sufficient number of circles the curve may be determined to any desired degree of accuracy.

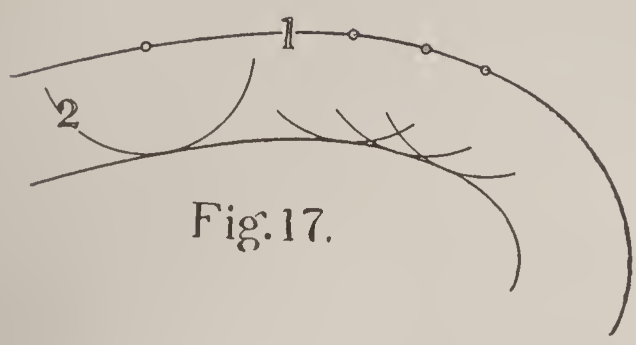


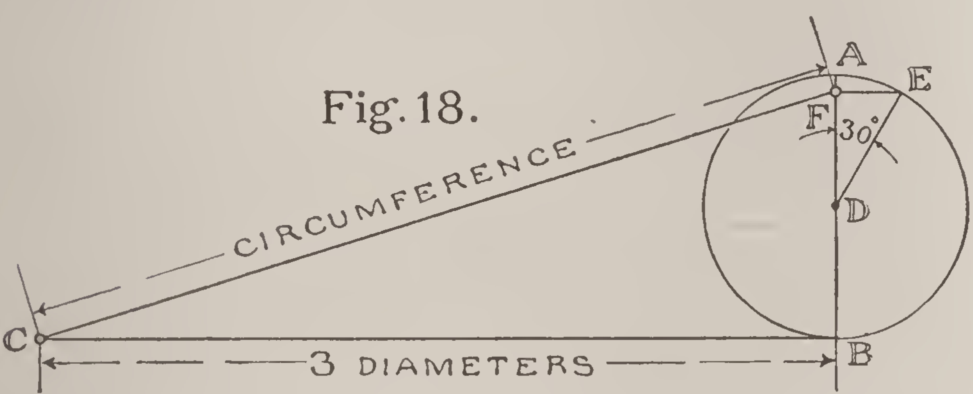
Fig. 17.

43. If the given curve is on tracing paper, to draw a parallel curve: Lay the tracing over two parallel lines, which are the required distance apart. Place the curve tangent to one line and trace the other. The required curve will be the envelope of the second line.

44. To draw a parallel to a given curve when its evolute is known: Find a series of centers on the evolute by means of which the given curve can be drawn with circular arcs. With the same centers and with similar arcs draw the required curve (§ 33).

45. To find the circumference of a given circle: Draw the diameter A B (Fig. 18) and from one extremity lay off three diameters on the tangent C B. Draw the radius DE making 30° with AB and draw EF parallel to CB. CF will be the circumference required. This is equivalent to $\pi = 3.1417 +$

Fig. 18.



46. If two plane curves roll on each other in the same plane a point attached to one will describe a roulette in relation to the other. The curve to which the point is attached is a generating curve and the other is a directing curve.

If the directing curve is a straight line and the generating curve is a circle the roulette will be a cycloid or a trochoid accordingly as the describing point is on or off the circumference. If both the directing and generating curves are circles, the roulette will be an epicycloid, or hypocycloid, or epitrochoid, or hypotrochoid, according to the conditions as shown in Fig. 19.

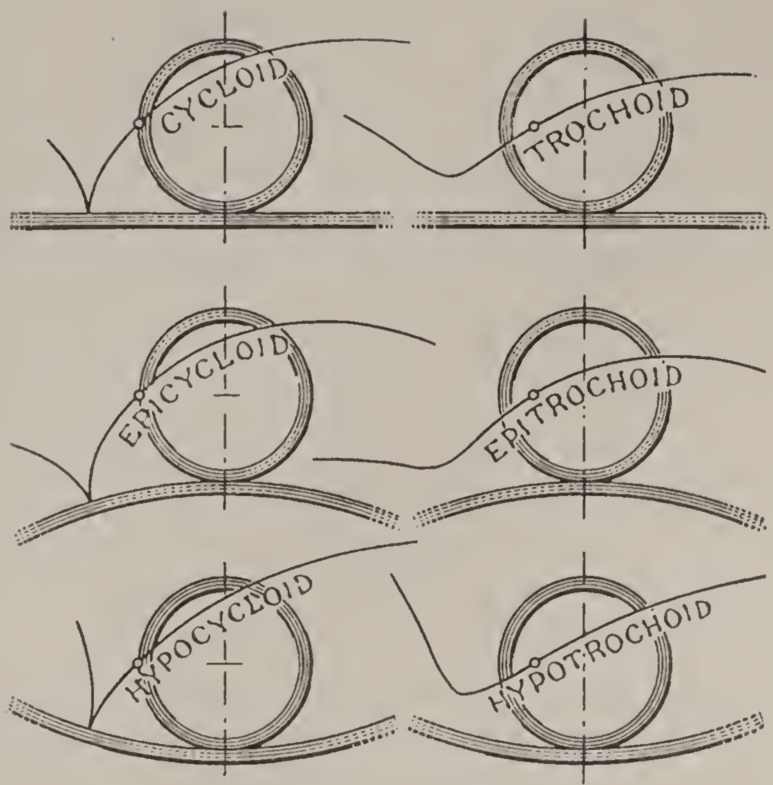


Fig. 19.

47. To draw a roulette: Let 1 (Fig. 20) be the directing curve, 2 the generating curve and *A* the describing point, one or the other of the curves being on tracing paper. On a strip of tracing paper, or, what is much better, a strip of transparent sheet gelatin, draw a straight line, 3, and lay off on it a distance *C D* of such length that when applied as a chord, *E F*, to the sharpest part of either curve the greatest distance, *G H*, from the chord to the curve shall not be greater than one eighth of the length of the chord. Lay off *C c* and *D d*, each equal to one fourth of *C D*. Place the two curves tangent to each other and to the straight line at the point *C*, the tracing of the straight line being uppermost. This is the position shown in Fig. 20. Prick the position of the describing point *A* and insert a needle at *C*. Turn both tracings about *C* as a center until the curves intersect at *D*. Insert a needle at *D*, remove the one at *C* and turn the tracing until the curves are tangent at *D*. This gives another position of the describing point *A*. By turning the line *C D* through half a circumference about *D* as a center the conditions will again be as shown in the figure, and the operation can be repeated.

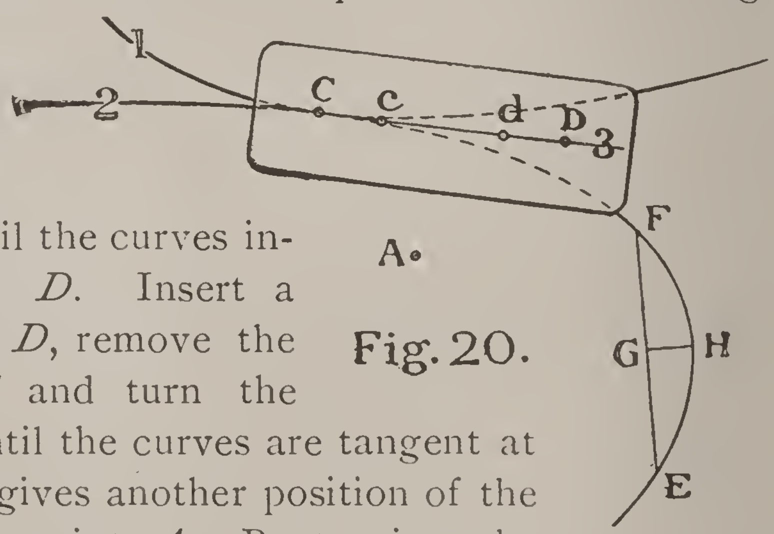


Fig. 20.

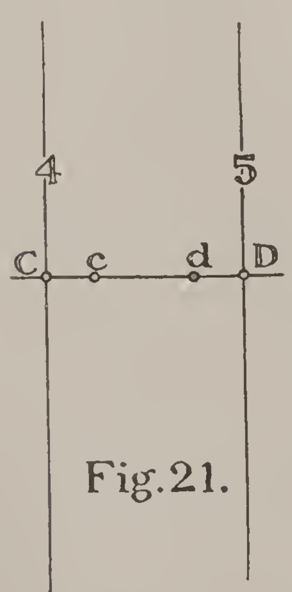


Fig. 21.

48. To draw a roulette when the generating and directing curves are circles or curves whose evolutes are known: Draw the lines 4 and 5, Fig. 21, at right angles to *C D*. If two curves are tangent to each other and to *C D*

at C their evolutes will be tangent to 4, or in case of circles, their centers will be on 4. Thus the curves can be placed accurately tangent to each other. In other respects the method will be as described in § 47.

CAMS.

49. A cam and its follower (Fig. 22) are links connected by higher pairing for the purpose of getting an arbitrary motion (§ 13). Certain objectionable features restrict their use to cases where the motion is difficult to produce accurately or even approximately by primary pairing (§ 11). The principal objection is that, owing to the smallness of the area of contact, a moderate force may produce a very severe pressure per square inch between the rubbing surfaces, and the result will be rapid wear (§ 14). Abrasion may be prevented by making the cam very large, thus slightly increasing the area of contact, or by inserting pieces of hardened steel at points where the wear is most severe (A, Fig. 22), or by substituting rolling for sliding contact (H, Fig. 25) (§ 15).

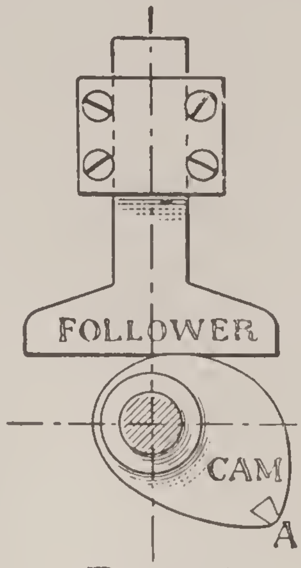


Fig. 22.

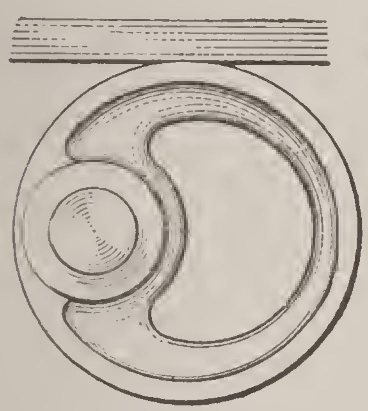


Fig. 23.

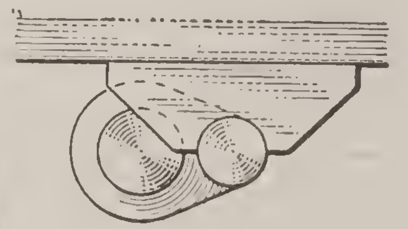


Fig. 24.

50. A circular cam with a flat follower, as in Fig. 23, will give a harmonic motion. The use of such a cam for that purpose, however, would most likely show bad judgment, seeing that primary pairs arranged as in Fig. 24 will give the same motion, and are at the same time more compact and capable of transmitting many times as much force without cutting.

51. Let A, B, C, and D, Fig. 26, rotate with equal uniform angular velocity in the direction of the arrows. Let their forms be such that the followers E, F, G, H are caused to move vertically with uniform velocity. Suppose the followers move through a distance a per revolution. Then A will be a circle whose circumference is a. B will be an involute of A. C will be an Archimedean spiral, whose polar equation is $r = a\ n$, where r is the radius vector and n its angular position in circumferences. D will be a parallel of the Archimedean spiral C, the distance between the parallel curves

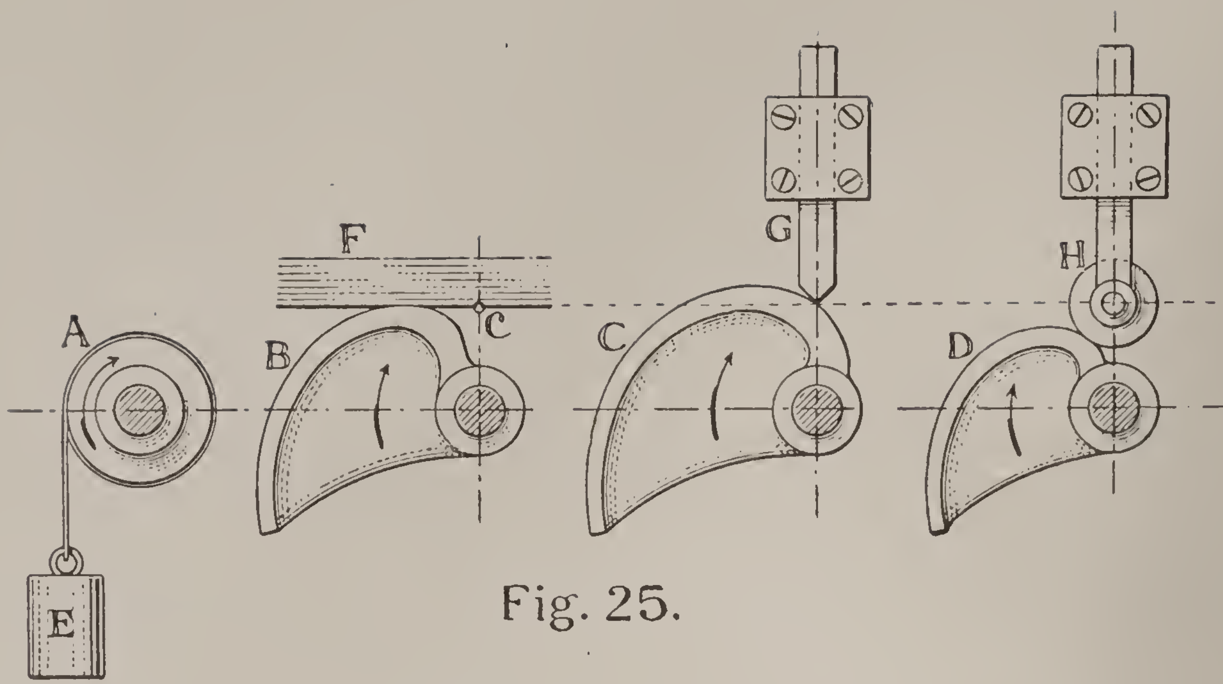


Fig. 25.

being equal to the radius of the roller *H*. The relation of the curves to each other will be rendered more apparent by noticing the manner in which they are generated. Let *A*, Fig 26, be a circle of the same diameter as *A*, Fig. 25. On the right angle *E b K* lay off *b c* = radius of *A* and about *c* as a center draw the circle *d* of the same diameter as *H* in Fig. 25. Suppose the right angle is rolled on *A* by the side *b E* without sliding, then the locus of *b* will be the involute of a circle, the locus of *c* will be an Archimedean spiral, and the envelope of *d* will be a parallel of the Archimedean spiral, the curves being equal in every respect to those in Fig. 25.

52. If the vertical motions of *E*, *F*, *G*, *H*, Fig. 25, are uniformly accelerated so that at each instant the space traveled per revolution is varying by an amount a per revolution, then *A* will be an involute of a circle. The diameter of the circle being $\frac{a}{\pi}$, *B* will be an involute of *A*. *C* will be the same as the relative path of a point *c* vertically over the center of rotation, and *D* will be a parallel of *C*. The polar equation of *C* in this case is $r = \frac{an^2}{2}$, where n is measured in circumferences.

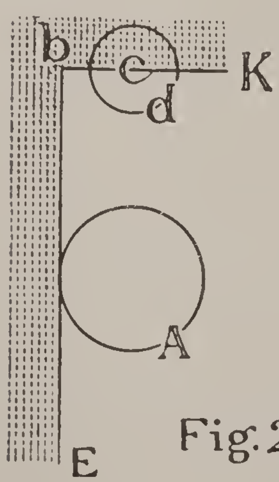


Fig. 26.

53. Whatever be the form of *A*, Fig. 25, then to produce the same motion, *B* will be an involute of *A*. *C* will have the form of the path of *c* in relation to *B*, and *D* will be a parallel of *C*. So that if any one of the forms *A*, *B*, *C*, *D* is known the others may be easily determined. And if the law of the motion is known, *C* may be determined by its polar equation.

54. The considerations which determine one's choice of an arbitrary motion will differ widely in different cases, and they lie for the most part outside of our present subject. It may be well, however, to mention one principle which has a somewhat general application in those cases where the work done, rather than the motion, is of primary importance.

A part of a machine will convey energy at a rate proportional to the force which it transmits. The strength of the part being constant it follows that to convey the maximum of energy the force should be constant. Thus, it may be desirable to design a machine in such a way that a certain part, the driving part for instance, shall be subjected to a constant stress.

Suppose a part A must of necessity act against a varying resistance, the resistance at each instant being a function of its position. A is driven by a series of links, among which is the link B. It is desired that B shall be subjected to uniform stress. Assume for the time being that B moves uniformly in the direction of the stress and then give A an arbitrary motion such that at each instant its velocity in the direction of its resistance is inversely proportional to the resistance. The velocity ratio being determined in this way, the stress on B will be uniform whether its motion is uniform or not.

In general, we may say that the rate of doing work should be constant. For example, if work is done against a constant resistance, as a friction or the weight of a body when lifted against gravity, then the velocity in the direction of the resistance should be constant. If the work consists in overcoming the inertia of a heavy mass then the motion should be uniformly accelerated. A cam is often used to close a clamp or vice. If the piece to be clamped is inelastic a spring resistance is inserted between the follower and the clamp. Here the motion should vary inversely as the space. It is easy to carry this principle too far. Thus various methods have been contrived to do away with the crank and connecting rod of the steam engine, the object being to equalize the tortional moment on the crank shaft. The object is desirable enough but there appears to be no easy way of accomplishing it.

55. Prob. 1. A cam like that shown in Fig. 22 rotates with uniform angular velocity. The follower is held in contact by its weight. The follower must rise with uniform velocity during two thirds of each revolution. Determine the form of the cam that may be run at the greatest number of revolutions without leaving contact

with the follower   Let the follower move $2\frac{1}{2}''$ and be $1\frac{1}{2}''$ from the center of the cam shaft at its lowest position.

56. The rising part of the cam will in this case be an involute of a circle (§ 51) and the falling part will be an involute of an involute of a circle (§ 52). For, since the follower drops by its own weight, that form of cam can be run at the greatest speed which allows it to drop with the uniformly accelerated motion of a free falling body.

57. The preceding paragraph will suggest one method of solving the problem, but the following solution is simpler and of more general application. Let *A*, Fig. 28, be at the center of a piece of tracing paper ($8'' \times 8''$) (§ 28). Through *A* draw twenty-four radial lines 15° apart, and number them 0, 1, 2, 3, etc., in such order that the rotation of the cam as indicated by the arrow will bring them successively to the same position. These lines can be most easily drawn with the 30° and 45° triangles. For by placing the triangles separately and together in various positions against the T square, all of the angles can be produced.

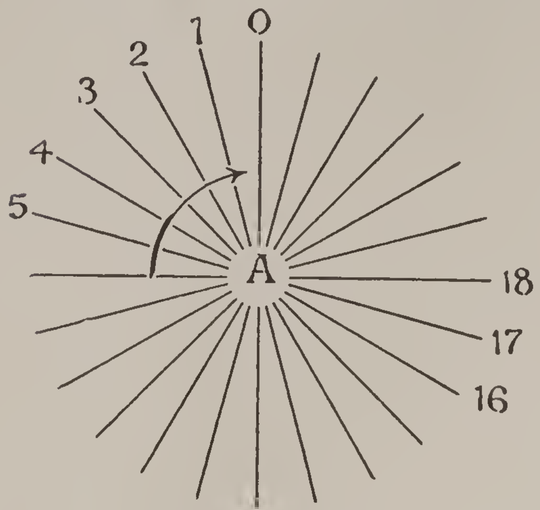


Fig. 28.

58. Let *A*, Fig. 27, be the center of the cam. The greatest radius of the cam will be $4''$, so that *A* should be about $4\frac{1}{2}''$ from the margin of the paper. 0 will be the lowest position of the follower and 16 its highest position. The dimensions are changed from those given in the problem to allow for a practical difficulty which will appear later. Divide the space between 0 and 16 into sixteen equal parts and make the divisions 0, 1, 2, 3, etc., beginning at the bottom. Lay a scale of equal parts diagonally across the lines

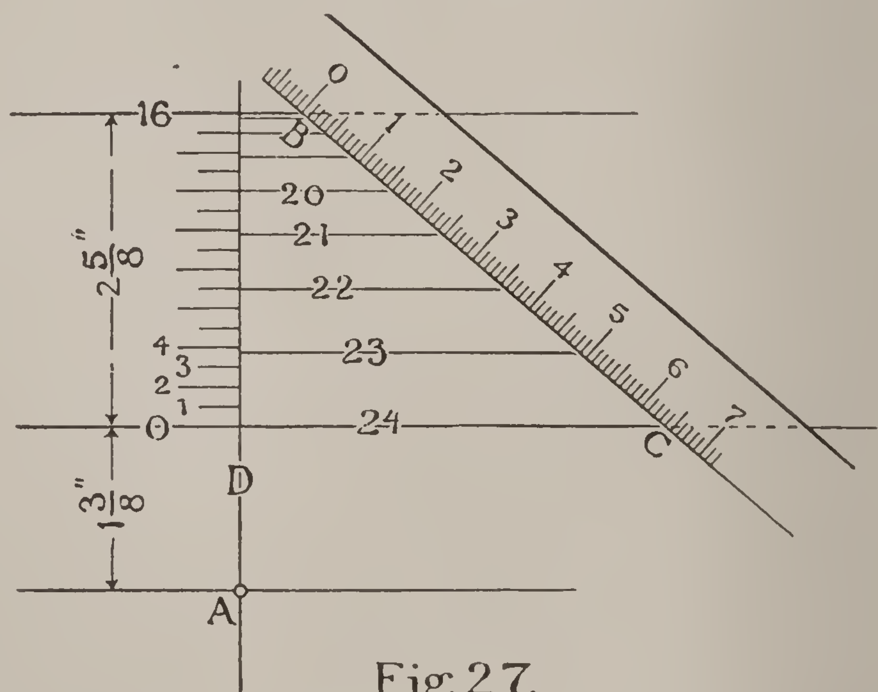


Fig. 27.

0, 16, at such an angle that there will be sixty-four divisions between the points of intersection *B* and *C*. Mark the position of the first division, counting from *B*, also the fourth, ninth, sixteenth, twenty-fifth, thirty-sixth, etc. Through the points thus laid off from the scale draw horizontal lines and number them 16, 17, 18, etc., beginning at the top (§ 69).

59. Lay the tracing, Fig. 28, over Fig. 27, so that the points *A* coincide, and, inserting a needle at *A*, turn the tracing about it as a center until 0 on the tracing coincides with *D*. Trace the line 0 corresponding to the lowest position of the follower. Turn the tracing through one division so that 1 coincides with *D* and trace the corresponding position of the follower, and so continue, alternately rotating and tracing through the twenty-four divisions. The envelope of the lines traced will be the required outline of the cam.

60. The outline will include a flat place, which is objectionable in practice, because the follower will strike it with a blow at each revolution. With *A* (Fig. 29) as a centre and a radius of 1½″, draw the circle *E* and substitute for the flat place a circular arc which is tangent to *E* at *D* and to the cam outline at *B* and *C*. The lowest position of the follower will then be 1½″ from the center of the shaft, as the problem requires. The drawing may be completed according to the dimensions in Fig. 29. The key-seat may be placed in any position except near *D*, where it would weaken what is already the weakest part.

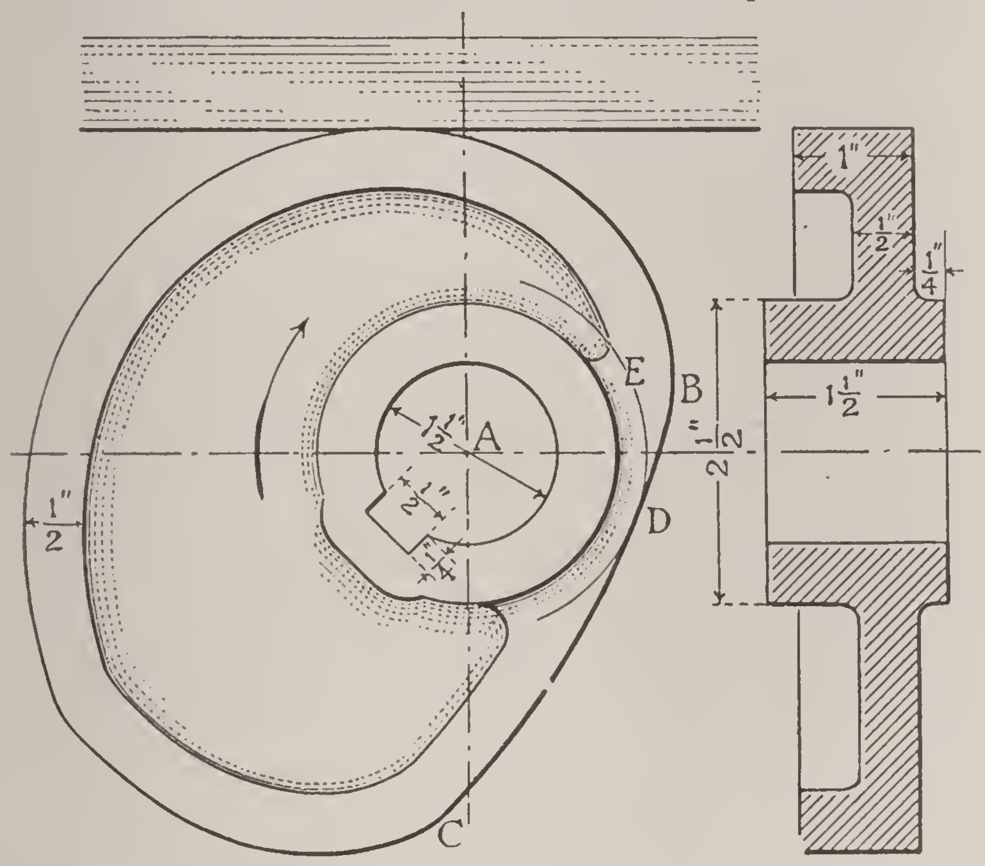
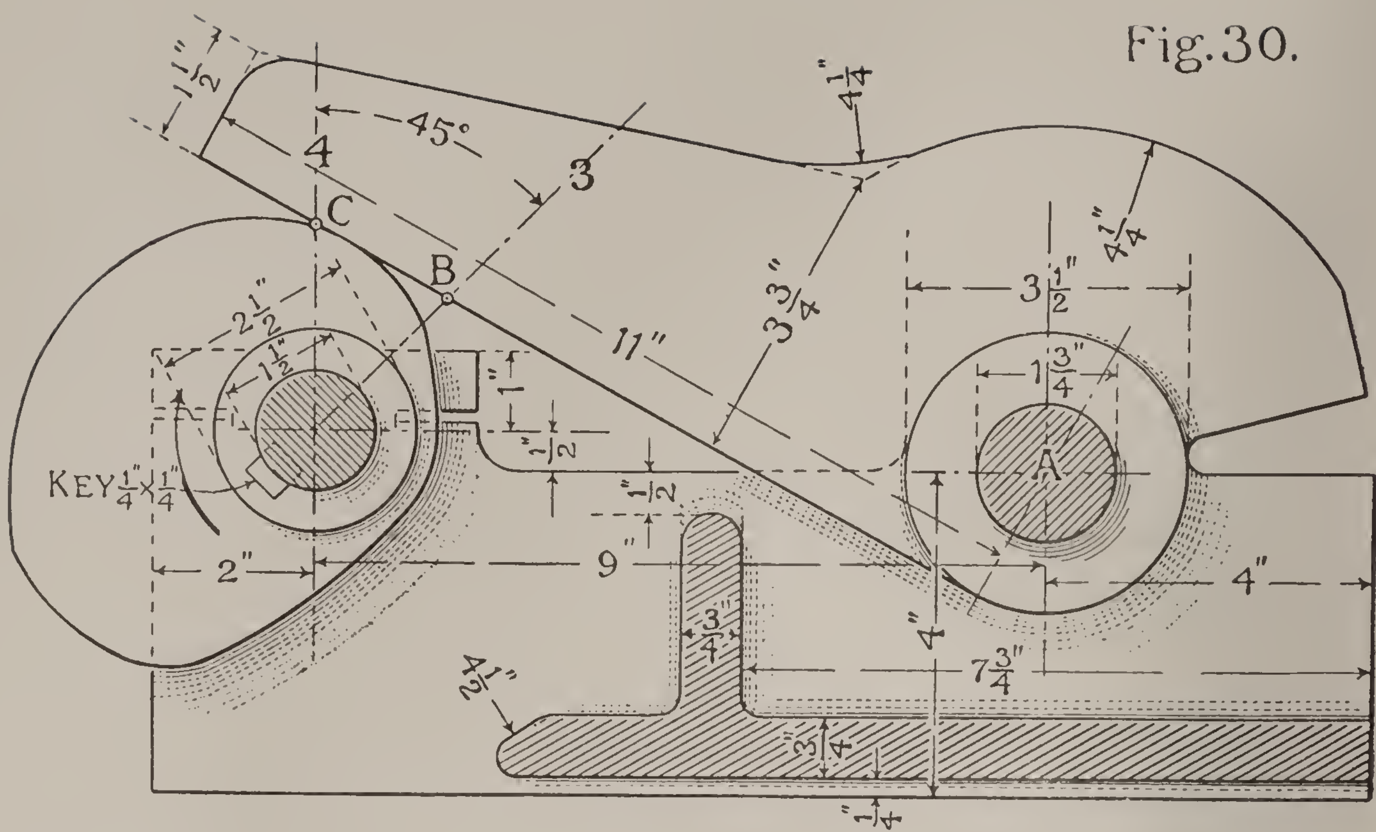


Fig. 29.

61. Prob. 2. Let the cam of the alligator shear, Fig. 30, rotate as indicated by the arrow, with uniform angular velocity. The fol-

Fig. 30.



lower remains in its lowest position during one sixth of a revolution and then rises, turning about the point A, during seven twelfths of a revolution, and falls during the remaining quarter. Let the follower rise with such angular velocity that the point of intersection, B, of its lower edge with the line 3 travels along the line 3 with uniform velocity, and let the fall be such that the point of intersection, C, travels along the line 4 with uniformly accelerated velocity. Make the greatest radius of the cam 4″ and the least radius 1½″.

62. Assuming the upward motion to be along the line 3 as above has the effect of increasing the angular velocity of the follower at the latter part of its stroke where the resistance is least (§ 54), and assuming the downward motion on the line 4 has the effect of increasing the angular acceleration at the latter part of the motion where the weight of the follower acts at a greater leverage.

63. On a piece of tracing paper (8″ x 8″) draw the twenty-four radial lines and number them as in Fig. 28 (§ 57). Assume the position F, Fig. 31, of the center of the cam shaft, taking care that there is room enough to complete the drawing to the dimensions given in Fig. 30.

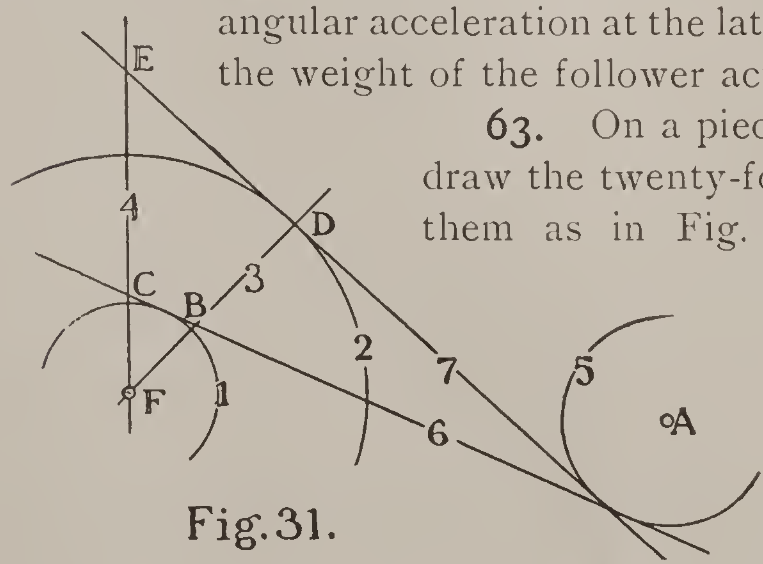


Fig. 31.

Draw the circular arcs 1, 2, with radii respectively equal to the least and greatest radius of the cam, and draw 3, 4, as in Fig. 30. Locate the center A about which the follower turns and draw the arc 5 with a radius equal to the perpendicular distance from A to the flat surface of the follower. 6 and 7, tangent to 5 and to 1 and 2, will be the extreme positions of the follower.

64. Since the revolution of the cam is divided into twenty-four intervals, fourteen of them, or seven twelfths of the whole, will be the period during which the follower rises, hence, divide BD into fourteen equal parts and number the divisions 0, 1, 2, 3, etc., beginning with 0 at the point B. The fall lasts one quarter of a revolution, or six of the twenty-four intervals, therefore, divide EC into six parts, and since the motion is uniformly accelerated, these parts will be proportional to the numbers 1, 3, 5, 7, 9, 11, increasing downwards (§ 69). To divide the line in this manner draw two parallel lines through its extremities, and across the parallel lines lay a scale of equal parts at such an angle that thirty-six of the parts fall between the two lines. Mark the required divisions along the edge of the scale and project them on the line EC by lines parallel to those already drawn. Number the divisions on EC 14, 15, 16, etc., beginning at E.

65. Lay the tracing over the drawing so that the focus of the radial lines coincides with F. Turn the tracing about F until its line 0 coincides with 4 and trace the line 6. Turn the tracing through one interval, making its line 1 coincide with 4, and trace a line tangent to 5 through the point 1. Turn the tracing another interval and trace a line tangent to 5 through the point 2, and so continue through the twenty intervals. For the four remaining intervals, during which the follower does not move, trace the line 6.

66. The envelope of the lines traced will include two flat places (§ 60), for each of which substitute a circular arc of 8″ radius. Complete the drawing to the dimensions given in Fig. 30.

67. Prob. 3. Fig. 32 shows a type of cam that one sees in the engine-valve gears of American side-wheel steamers. The crank A turns about the center B with uniform angular velocity. The connecting rod C is so long that its angularity may be neglected. The cam is rigidly connected to the rocker-arm D, which swings to and fro through an angle of 90°. M and N are the extreme positions of the follower. During half a revolution of the crank, while

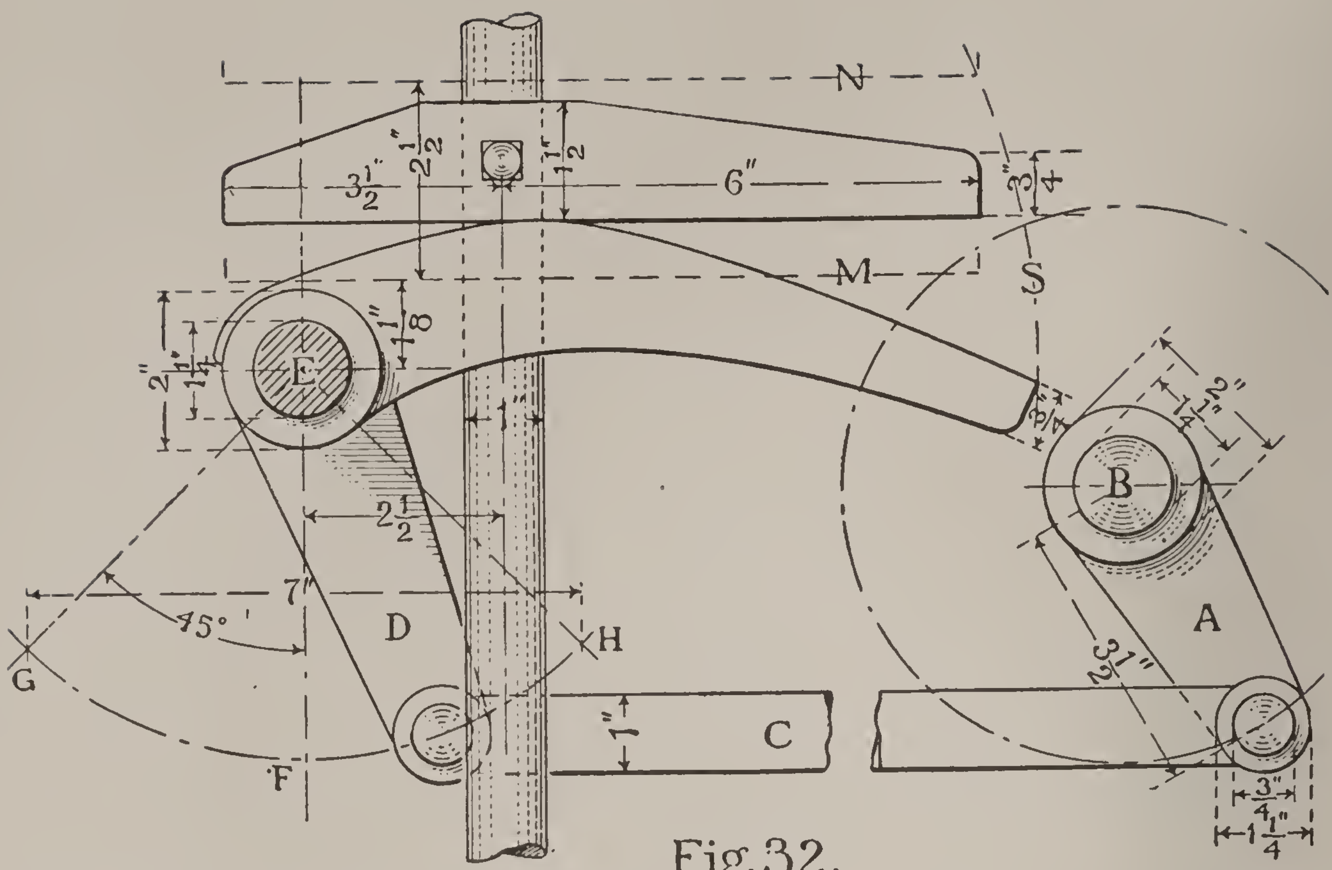


Fig. 32.

the rocker-arm *D* swings from *F* to *G* and back to *F*, the follower remains stationary in its lowest position, *M*. During a quarter of a revolution, while *D* swings from *F* to *H*, the follower rises to the position *N*, and during the remaining quarter it falls back to *M*.

68. It is desired to make the motion of the follower uniform. But a reciprocating motion of uniform velocity can only be approximated, and the closer the approximation the greater will be the stress and shock of starting and stopping, it being physically impossible for any mass to pass from a state of rest to a state of motion without passing through an intermediate state of acceleration. There is also the difficulty that the cam moves with infinite slowness at the end of its stroke, and hence would need to be of infinite length to give the follower a finite velocity. We will, therefore, make the following assumption in regard to the motion of the follower: If the period during which it rises be divided into twelve equal intervals then the motion shall be uniformly accelerated during the first two, remain constant during the next eight, and be uniformly retarded during the last two.

69. If a body starting from rest is uniformly accelerated in the direction of its motion, and the motion is divided into equal intervals of time, then the initial velocity will be 0, and if the space covered during the first interval is 1 the velocity in space per interval at the end of the first interval will be 2, the space covered during

the second interval will be 3, and at its end the velocity will be 4, the space for the next interval will be 5, and its final velocity 6, etc. The spaces covered during successive intervals being represented by the numbers 1, 3, 5, 7, etc., the total space for n intervals will equal n^2. If a body is uniformly accelerated from rest during two intervals, and after that the motion is uniform, the spaces covered during successive intervals may be represented by the numbers 1, 3, 4, 4, 4, etc. If the period of acceleration is divided into three intervals the numbers will be 1, 3, 5, 6, 6, 6, etc. If into four, 1, 3, 5, 7, 8, 8, 8, etc.

70. Locate the point E, Fig. 33, of the center of the rock-shaft, and draw M and N as in Fig. 32. Divide the space between M and N into twelve parts proportional to the numbers 1, 3, 4, 4, . . 4, 3, 1, and number the divisions as in Fig. 33. On a piece of tracing paper (8″ x 14″) locate the point E, Fig. 34, about 4″ from the left end and 2″ from the upper edge, and draw Eo in direction at right angles to the length of the paper. With center at E and radius of 3½″ draw the arc O of 90° to represent one quarter of a revolution of the crank A. Draw T parallel to Eo and R at 45° and through the point of intersection, 12, draw the arc P. Divide O into twelve equal parts and project the points of division on P by lines parallel to Eo.

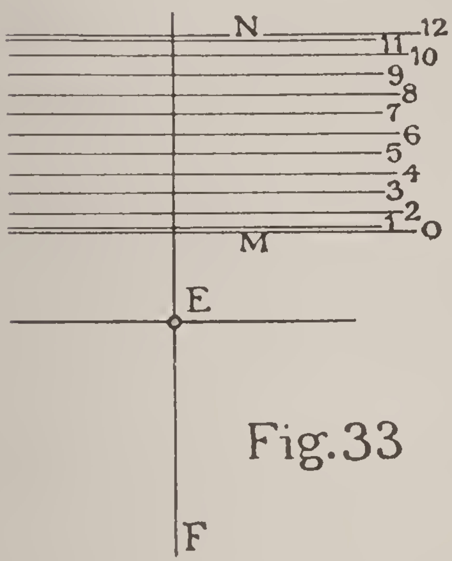


Fig. 33

Number the divisions as in the figure and through each one draw a radial line to the center E. The radial lines will represent successive angular positions of the cam corresponding to the twelve positions of the follower in Fig. 33. Lay the tracing over Fig. 33 so that the points E coincide and Eo coincide with EF. Trace the line M and turn the tracing about E so that E1 coincides with EF and trace the position of the follower marked 1, and so continue till all the positions of the follower are traced. The cam may be cut off at a point determined by the arc S, Fig. 32, whose

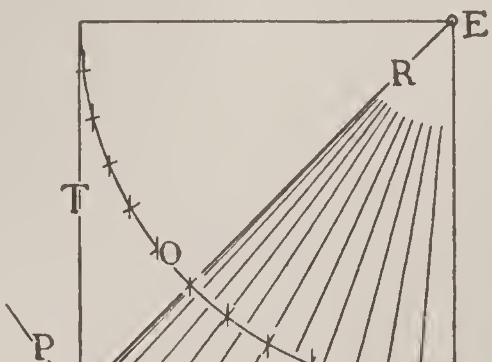


Fig. 34.

center is at E, and the lower edge of the cam should be drawn with a curve that gives it a gradual taper.

71. In Fig. 35 suppose the sphere A is held in contact with the horizontal plane by an active force such as gravity, and suppose the sphere B touches two parallel planes. In both cases there is one degree of constraint (§ 3), but they differ in that A is constrained only so long as a force keeps the bodies in contact, while B is constrained by virtue of its form, or, rather, by a resistance to a change of form. Resistance to a change of form we may call a passive force since it acts only when acted upon. In the one case, constraint is inseparable from the idea of force, while in the other the idea of force, as well as force itself, may be eliminated.

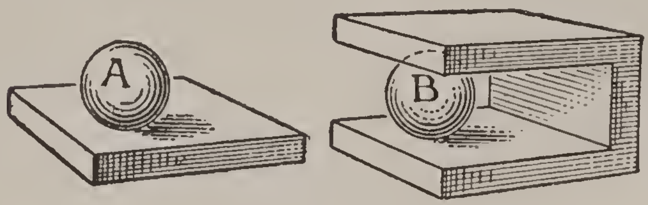


Fig. 35.

72. A pair of elements having five degrees of constraint is said to be closed. If the constraint is wholly due to the forms of the elements, the pair is form closed. If a force is necessary to the constraint, the pair is force closed. Cams are commonly spoken

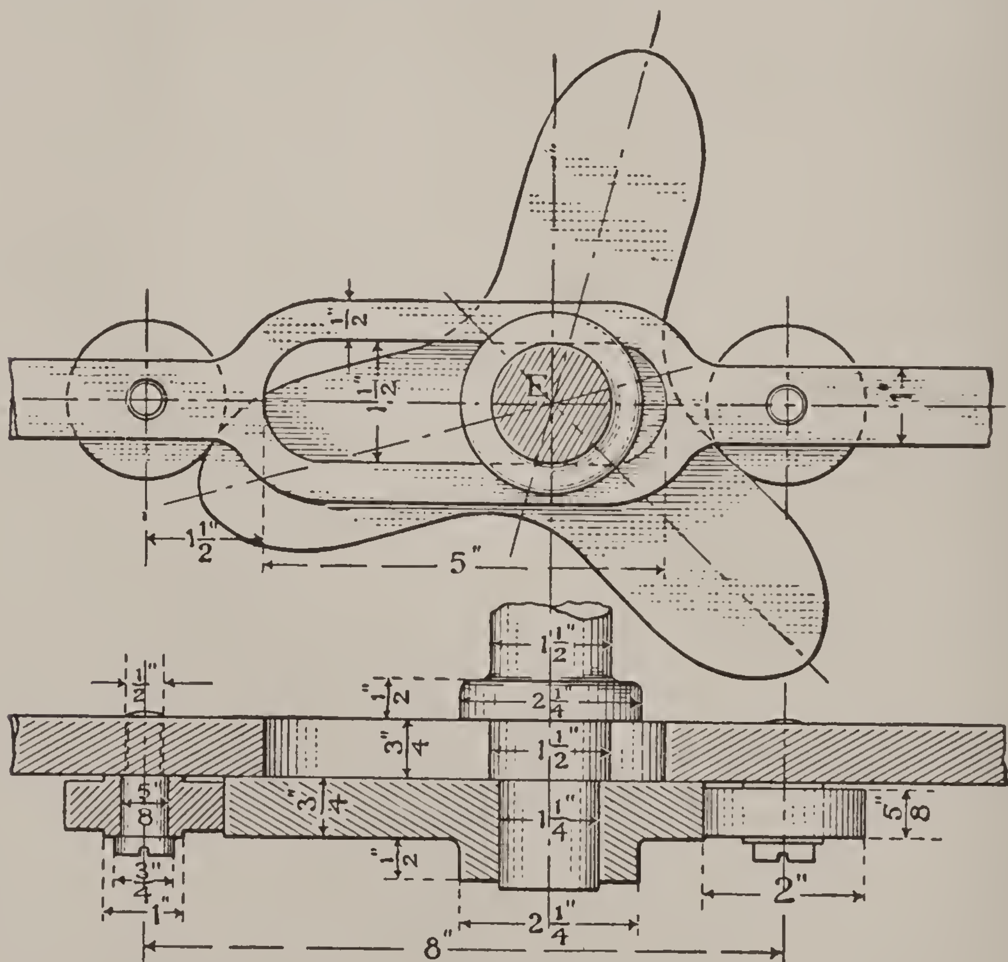


Fig. 36.

of as being covered or uncovered according as the pair is form or force closed. For example, the cams that have been considered thus far are uncovered.

73. If a machine contains a link that is driven through a force closed pair, and the motion of the link is at any time accelerated in the direction of the closing force, then there is a limit to the speed at which the machine can be run depending on the magnitude of the closing force. For high speeds, or speeds which are indeterminate, such links should always be driven through pairs that are form closed.

74. Prob. 4. The covered cam, Fig. 36, rotates with uniform angular velocity. The follower moves $3''$ with a harmonic motion of three double vibrations per revolution. The diameter of the rollers is $2''$ and the distance between their centers is $8''$. Determine the form of the cam.

75. When the center of one roller is at its least distance from the center of the cam the other will be at its greatest distance. The sum of the two distances will be $8''$ and their difference will be the travel $= 3''$. Hence, the least distance will be $2\frac{1}{2}''$. Assume E, Fig. 37, as the center of the cam-shaft, and on the center line of the follower E, A locate the points 0, 12, which are the extreme positions of the center of one of the rollers. Draw the semicircle $3''$ in diameter with center at 6, and divide its arc into twelve equal parts. Project the divisions on the line EA and number them as in the figure.

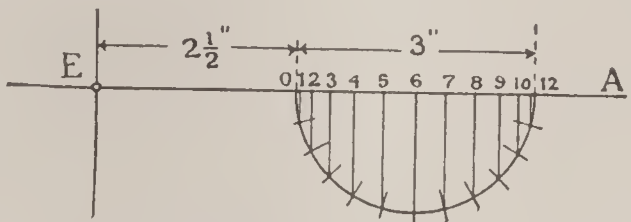


Fig. 37.

76. Draw on tracing paper the lines $E0$ and $E\,12$, Fig. 38, at an angle of 60°, and divide the angle into twelve equal parts by the lines $E1$, $E2$, etc. Lay the tracing over Fig. 37 so that the points E coincide and $E0$ coincides with EA. On the tracing draw a circle of $1''$ radius and center at 0, Fig. 37. Turn the tracing so that $E1$ coincides with EA and draw another circle of the same radius and center at 1, Fig. 37, and so continue. The envelope of the several circles will be a curve which, repeated

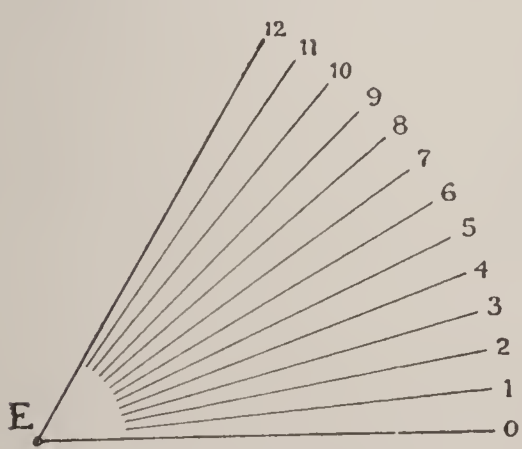


Fig. 38.

six times, is the outline of the cam. Three of the repetitions will be with the tracing turned face down.

77. In the last problem we assumed certain dimensions and a harmonic motion for the follower. Those assumptions could have been anything else, subject to the following limitations: The rollers must be of the same diameter. Their centers and the center of the cam shaft must be in the same straight line, and only half of the motion can be arbitrary. If the motion of the follower during half a revolution of the cam is assumed, then its motion during the remaining half must be a repetition of the first half, in the same order, but in the opposite direction. Thus, if the motion is accelerated at a certain rate from left to right during the first half, then it must be accelerated at the same rate from right to left during the second half.

78. Covered cams that are subject to none of the above limitations can be made as illustrated in Fig. 39. Each pair has two contacts and two irregular surfaces. One irregular surface is always

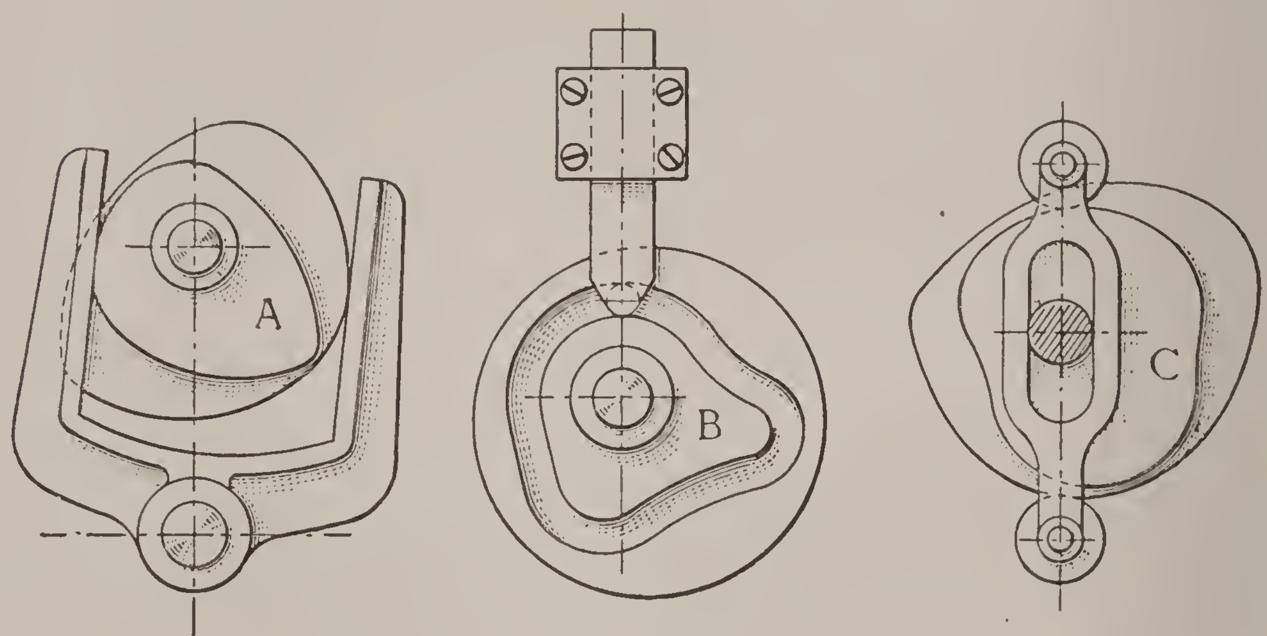


Fig. 39.

deducible from the other (§ 13). Thus in *A* and *B* the outlines are simply parallel curves. In *B*, by making the pin tapering and the groove of trapezoidal section, the lost motion resulting from wear can easily be taken up.

79. Prob. 5. Fig. 40 represents an open and cross belt reversing gear. The driving shaft *A* has a single wide pulley and the driven shaft *B* has two loose pulleys and two fixed pulleys. The loose pulleys are placed on the outside, because their bearings must

be oiled. By shifting the open belt the shafts will run in the same direction, and by shifting the cross belt they will run in opposite directions.

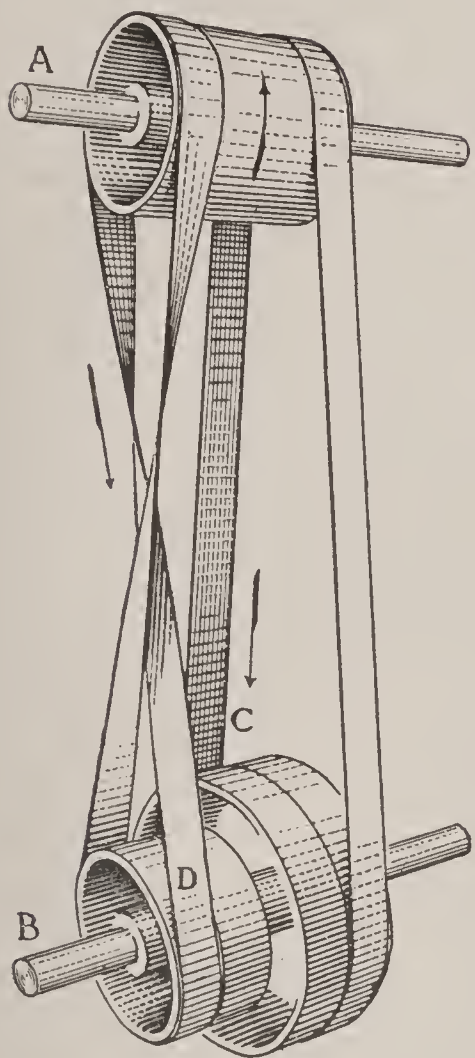


Fig.40.

Such a device is used for driving metal planing machines, and to get a quick return motion the pulleys on the shaft B are made of two sizes. During the forward or cutting stroke of the planer the shaft B is driven through a large pulley which gives a slow speed, and during the return stroke, where no useful work is done, the shaft is driven through a small pulley.

80. There is needed a device, to be placed near the shaft B, for shifting both belts.

It must operate on the belts at the points C, D, where they approach the pulleys.

One belt must be shifted at a time, since both belts must never be on the fixed pulleys at the same time.

The device must be operated by a single motion of a link which can be driven by the planer itself, thus making the whole operation automatic.

An adjustment must be provided whereby the necessary motion of this driving link can be varied. For, since the first belt to be shifted will always be the one that, for the time being, is driving the machine, it follows that the momentum of the machine must be depended upon to shift the second belt. The adjustment is to provide against the uncertainty as to how far the momentum will carry the machine after the first belt is shifted.

81. Fig. 41 represents a belt shifter that answers to the above conditions. It is a covered cam with two followers. The cam turns about the point A and the followers turn about the points B, C. When the cam is in its middle position, as represented in the figure, the portion of the slot which is between the two rollers is irregular in form and is the only portion that moves the followers. The rest of the slot is simply made up of circular arcs, with A as a center, and serves to hold either follower in a fixed position while the other

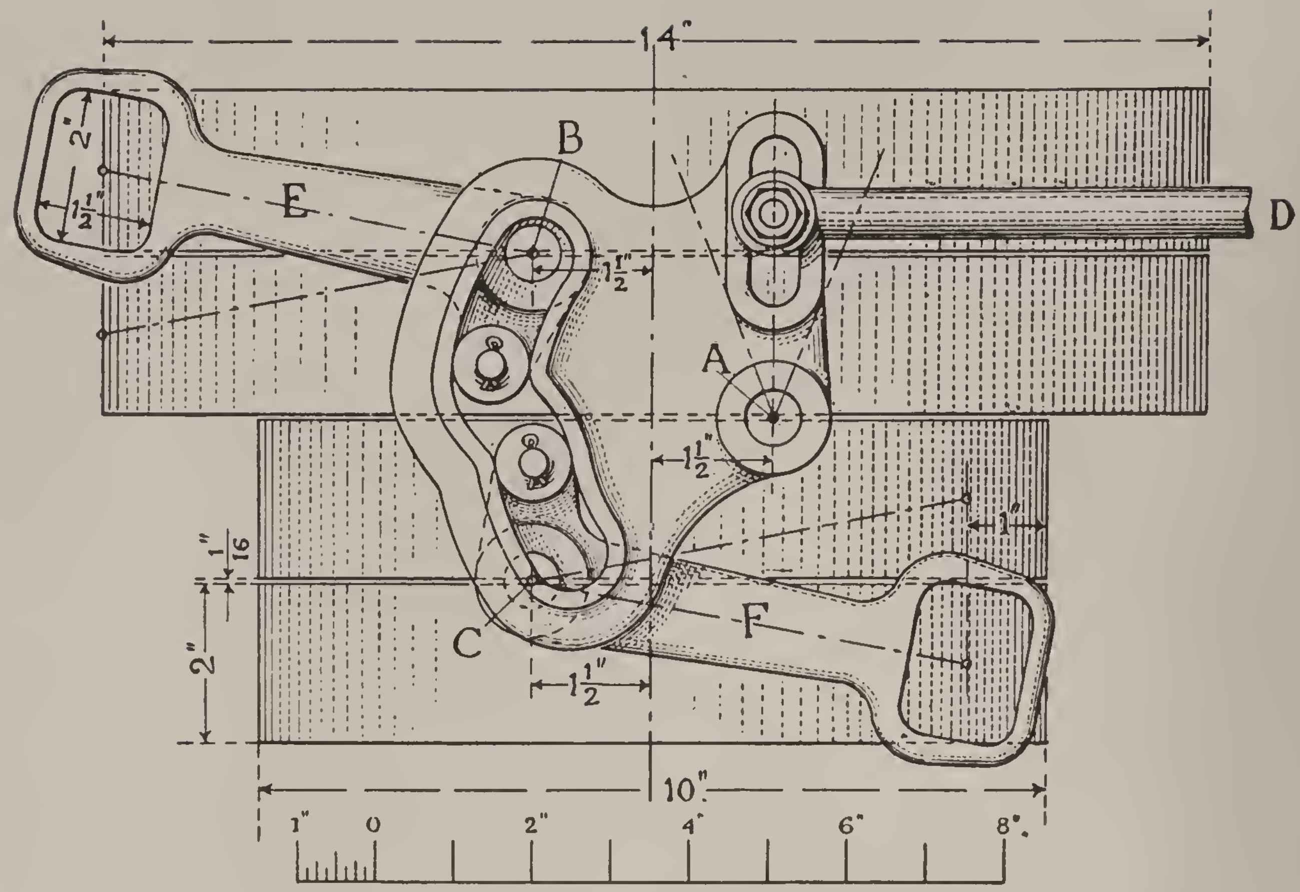


Fig. 41.

one is moving. Thus if the rod *D* is thrown to the right the lever *E* is moved and *F* is held fixed. Throwing the rod to the left moves *F*, and *E* remains fixed.

The center of the belt hole in the lever *F* is placed 1″ from the edge of the pulley to allow for the angularity of the cross belt.

82. Locate the points *A*, *B*, *C*, Fig. 42, by the dimensions given in Fig. 41. Draw the circular arcs 1, 2, that are described by the centers of the rollers, and locate the extreme positions 3, 4, of the

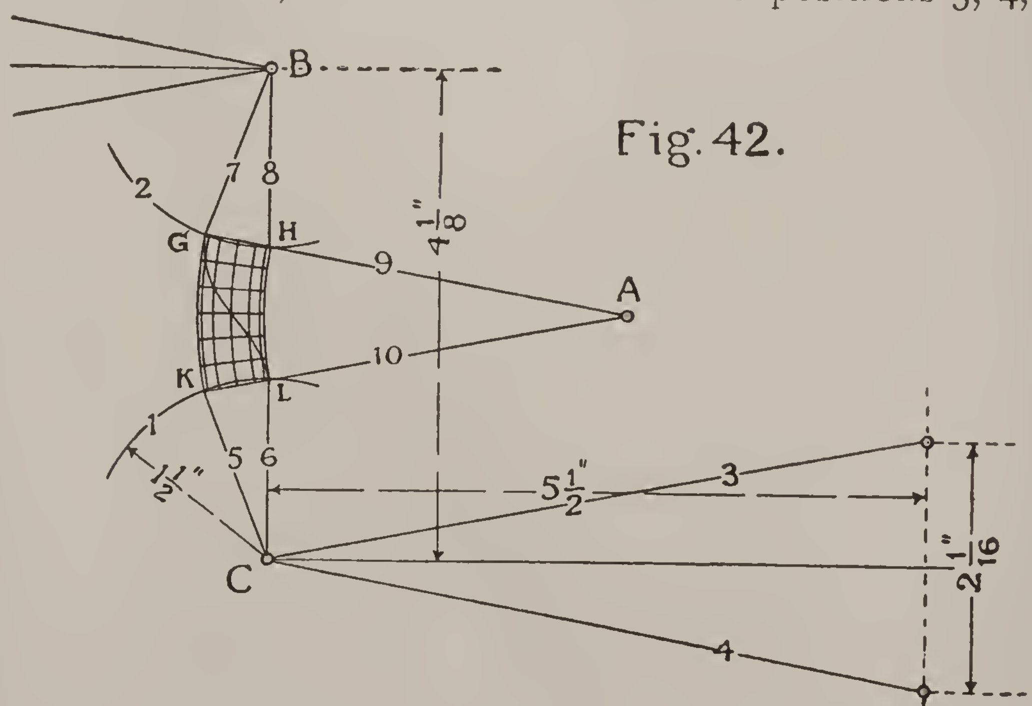


Fig. 42.

center line of the shifter F. Make the angles 5,6 and 7,8 equal to the angle 3,4. Then the points of intersection, G, H, K, L, will be the extreme positions of the centers of the rollers.

83. The point A is so placed that the line 9 passes through the points GH, and the line 10 passes through the points K, L, a condition that is unessential, but it renders the motions of the shifters as symmetrical as they can be made. By symmetrical motions we mean that the two shifters move alike, and each one moves the same in either direction. Perfect symmetry is impossible, because the arcs 1, 2 are curved in opposite directions.

84. Divide the angle 9, 10 into six equal parts by radial lines, and divide the distance GH into six parts proportional to the numbers 1, 3, 5, 5, 3, 1. With the center at A draw circular arcs through the points of division. The intersections of the circular arcs and the radial lines will determine a curve, GL, which is the path of the centers of the rollers in relation to the cam.

The rollers are 1″ in diameter. Hence, the outline of the cam slot will be the envelope of 1″ circles whose centers are on the curve GL.

85. The total angular motion of the cam will be twice the angle 9,10. The total angular length of the slot, measured from center to center of the terminal semicircles, should be 67½°, or a trifle more than three times the angle 9,10. This gives a slight clearance between the roller and the end of the slot at each of the extreme positions of the cam.

The stud on which turns the driving link D can be fixed at any point in the radial slot. Thus the distance that D must move can be adjusted.

Finish the drawing to the dimensions given in Fig. 41. Parts that are not figured may be measured by the attached scale.

86. In all the preceding problems the follower moves in a plane at right angles to the axis about which the cam rotates. When the follower must move in a direction parallel with the axis of the cam shaft a crown cam, Fig. 43, is used.

The two forms occur with about equal frequency in practice, although the latter, unless formed on a milling machine, is more difficult to make, on account of having a warped surface. The crown cam is also less easily defined in a drawing.

87. Prob. 6. Design a covered crown cam, Fig. 43, that gives one double harmonic vibration to the follower for each revolution of the cam. Let the amplitude of the vibration be 1″.

88. The form of the slot is most easily represented on the developed cylindrical surface of the cam. The workman can then wrap the development around the cylinder, and with a prick punch mark any number of points in the curve.

Let the lower face of the cam be finished, so as to give a convenient reference line for locating the curve. Draw *AB* equal to the circumference of the cam (§ 45), and draw the circle 2 of a diameter equal to the amplitude of the vibration. Divide the circumference of the circle 2 and the line *AB* into the same number of equal parts. The intersections of the horizontal and vertical lines

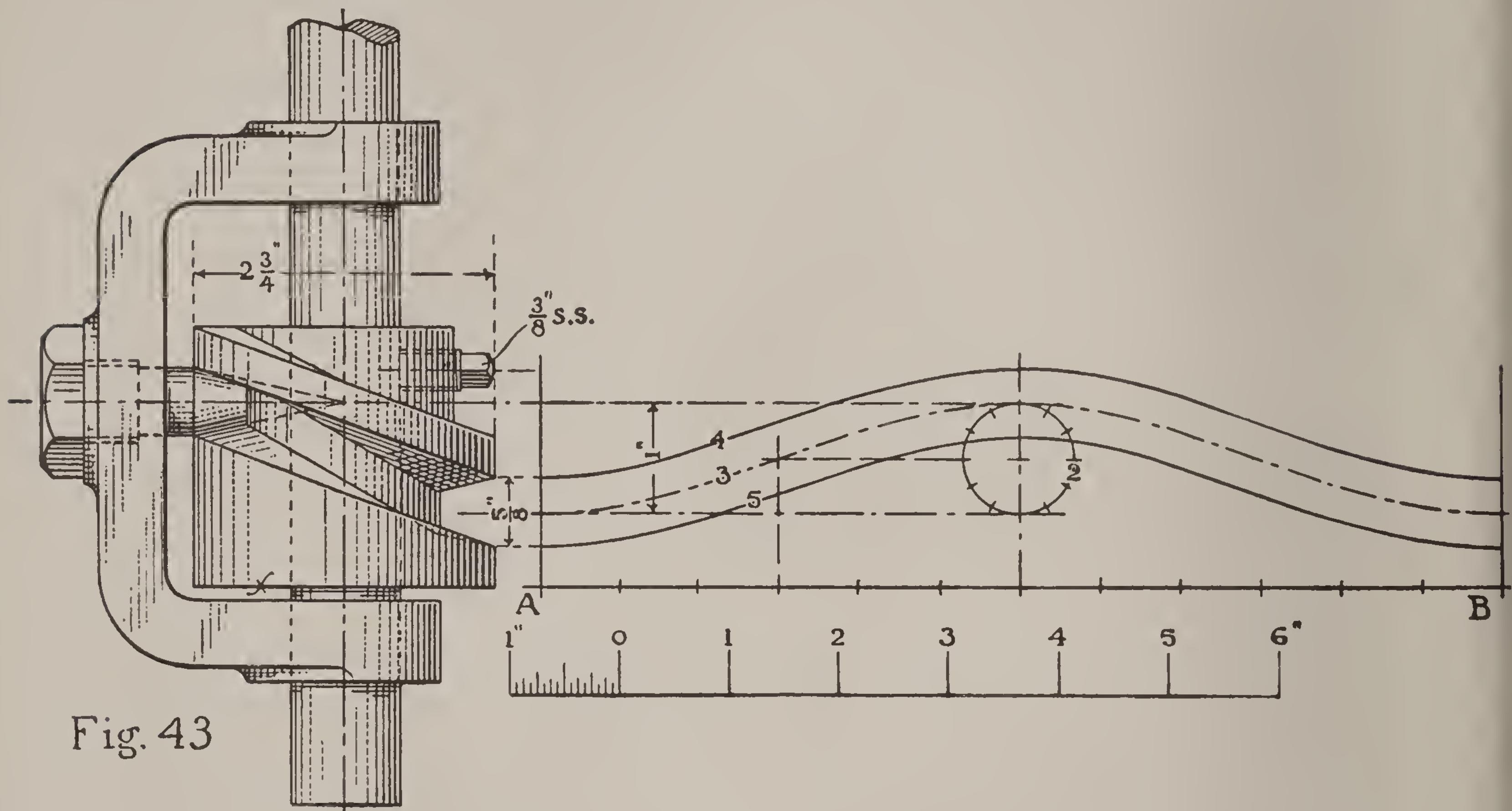


Fig. 43

through the points of division will give a sine curve 3, which is the development of the center line of the slot. The developed edges of the slot will be the parallel curves 4 and 5 (§ 42).

89. In this particular case the center line of the slot is an ellipse in space, and by turning the cam to a certain position, that shown in the figure, the ellipse will be projected in a straight line, and the edges of the slot will be projected in lines that are very nearly straight.

90. A right section across the slot has the form of a symmetrical trapezoid, and the element of the follower is a conical surface, to provide for taking up any lost motion that may result from wear.

The adjustment is made by shifting the cone in the direction of its axis.

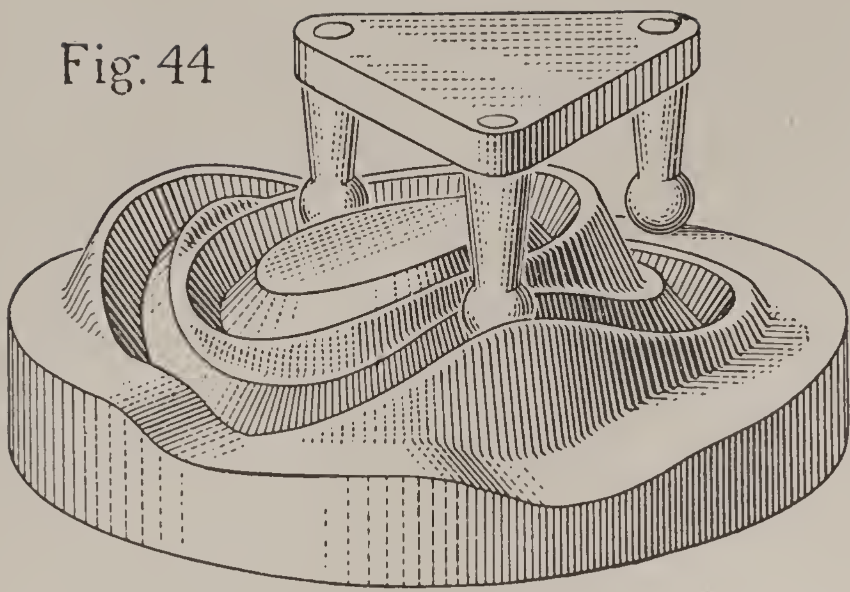
The vertex of the cone should be in the axis of the cam. Although it is desirable, for the avoidance of friction, that the cone should be as sharp as possible; if it be made sharper than the above there may occur a condition when it will result in a reduction of the wearing surface. Such a condition occurs when the length of the least radius of curvature of the center line 3 approaches the half width of the slot. Under these conditions an edge of the bottom of the slot may be a curve which crosses itself. The double cusp, formed by the curve crossing itself, can have no real existence in the cam; hence the reduction of the wearing surface.

91. If the direction of the desired motion is neither normal nor parallel to the axis of the driving shaft, the motion can still be obtained by a single cam; though, in practice, it is generally simpler to get the motion by a combination of two or more cams, as we are thereby enabled to resolve the motion into components, and consider each component separately. A similar method can be used when the motion is defined by complex curves in either two dimensional or three dimensional space.

92. The following illustration will make evident the possibility of getting any arbitrary motion by means of two elements. Let one element consist of three equal spheres rigidly connected together, and let the other element consist of two tubes and two parallel surfaces, also rigidly connected together, the inside diameter of the tubes and the distance between the parallel surfaces being each equal to the diameter of the spheres. With a sphere in each tube, and the remaining sphere between the parallel surfaces, we have a pair of elements with five degrees of constraint, for each of the tubes offers two degrees of constraint, and the parallel surfaces offer one degree. The axes of the tubes can be any two curves in space, and the parallel surfaces can be of any form; hence the motion may be of the most general character. A third tube may be substituted for the parallel surfaces; but the curve of its axis will be partly determined by the axes of the other two. Observe that with three tubes there will be line contact between the elements.

It follows, therefore, that any arbitrary motion may be constrained by two elements having three lines of contact.

Fig. 44



93. By introducing force closure, the elements may be greatly simplified, as in Fig. 44, which represents a form of cam that can be made to give an arbitrary motion of the most general character.

GEARING.

94. A pair of gears are two bodies which are capable of driving each other by means of projections from the one engaging with recesses in the other. Leaving out of consideration, for the present, a few forms which admit of only point contact, we will consider the general theory of those which admit of line contact.

95. When, as with the line, there is more than one point of contact, a definite relation must connect the velocities of the points, lest a fast-moving point push apart the contact at a slow-moving point. This relation does not concern the actual motions of the points, but only those components of the motions which are normal to the surfaces in which the points lie, since motions that are tangent to the surfaces at the points of contact can have no effect in separating them. Consider, now, that the projections and recesses are reduced to infinitesimal dimensions. What was before a tangent surface at the point of contact, becomes now a tangent line, and any tangential motion that does not separate the contacts must take place along this line. The only lines that can slide on each other and remain in contact throughout their length are lines of equal and constant curvature, and of these there are only three pairs: the straight line sliding on a straight line, the circular arc on a circular arc of equal radius, and the helix on an equal helix.

But in a pair of gears we have a relative motion besides the sliding, else the one could not drive the other. Of the three pairs cited above, only one admits of a relative motion other than the sliding, and that is the pair of straight lines, for a rigid straight line

can revolve about itself and at the same time slide on a coincident line, and this is true neither of the rigid circular arc nor of the rigid helix.

In our pair of gears with infinitesimal teeth the line of contact must therefore be a straight line, and the working surfaces of the gears will be ruled surfaces or surfaces generated by the motions of a straight line. It follows, therefore, that if gears are to have line contact, their teeth must be built on ruled surfaces.

These surfaces are called axodes or pitch surfaces, and we need only learn the capabilities of such surfaces to be able to predict all the possible forms of line contact gearing.

96. A body may be located in a plane by locating two of its points. Let A, B and A', B' be two positions successively occupied by a body in a plane. A can move to A' by any one of an infinite number of circular arcs all lying in the plane, and the locus of their centers will be a straight line bisecting AA' at right angles. Similarly with BB'. At the intersection of the two loci will be a point O, about which A can rotate to A' and B to B'. Hence any change of position in a plane may be produced by a simple rotation about a fixed point in the plane. An apparent exception to this is the particular case where the motion is one of simple translation. This need make no difficulty, however, the difference being merely that here the center O is at infinity.

97. Applying the same reasoning to a spherical surface, we have the following: Any change of position in the surface of a sphere may be produced by a spheric rotation about a fixed point in the surface of the sphere. But any motion in the surface of a sphere is also a rotation about the center of the sphere; it can, therefore, be produced by a simple rotation about a line joining the center with the point O in the surface.

98. From the above, it follows that, "if a rigid solid body move in any way whatever, subject only to the condition that one of its points remains fixed, there is always one line of it through this point common to the body in any two positions."*

99. Consider, now, the most general motion of a rigid body of which no point is fixed. Let A and A' be one point of the body in any two positions. By a simple translatory motion, bring the point A' back to its first position A. The first and last positions have the point A in common. There is, therefore, some line passing

* Euler's theorem. See Thomson and Tait: *Treatise on Natural Philosophy*, Vol. I, p. 69.

through the point A, about which the body can be rotated from the first position to the last. During the translation, this line remained parallel to itself. Any change of position of a body can, therefore, be produced by a rotation and a translation. But the translation can be resolved into two components, one of which is parallel to the axis of rotation, and the other perpendicular thereto. The perpendicular translation and the rotation, being in the same plane, can be produced by a single rotation (§ 96). Therefore, any change of position of a body can be produced by one rotation about a fixed axis and one translation in the direction of the axis.

100. Let any relative motion be resolved into an infinite number of infinitesimal rotations and translations of the kind assumed above, and it follows that any motion can be produced by a combined rolling and sliding, the sliding being always in the direction of the instantaneous axis of rotation.

Any motion can, therefore, be produced by the combined rolling and sliding of ruled surfaces having straight line contact.

Hence any motion can be produced by a pair of line contact gears. Thus it is theoretically possible to attach a gear to each element of the cam shown in Fig. 44, so that for the motion allowed by the cam the gears will work together correctly and with line contact.

101. Ruled surfaces are conveniently divided into four classes: the undevelopable warped surface, the developable warped surface, the cone, and the cylinder. The combinations which satisfy the requirements for pitch surfaces are eight in number. A pair of undevelopable warped surfaces may have rolling and sliding contact or rolling contact alone. The same is true of a pair of developable warped surfaces and of a pair of cylinders. The two remaining combinations are: a pair of cones with rolling contact, and a cone and a developable warped surface with rolling and sliding contact. Each of the resulting motions can be cyclical; that is, the gears can be brought back to a previous position without reversing.

102. For most purposes gears are made to revolve about fixed axes with a constant velocity ratio. When this is the case the ruled surfaces must also be surfaces of revolution. Only five out of the eight combinations can be made to answer this condition.

103. The undevelopable warped surfaces, which both roll and slide, can be hyperbolas of revolution, and, as such, they are the

true pitch surfaces of the skew gear, the worm gear, and, with one exception, of all forms of line contact gears having fixed axes that are neither parallel nor in the same plane.

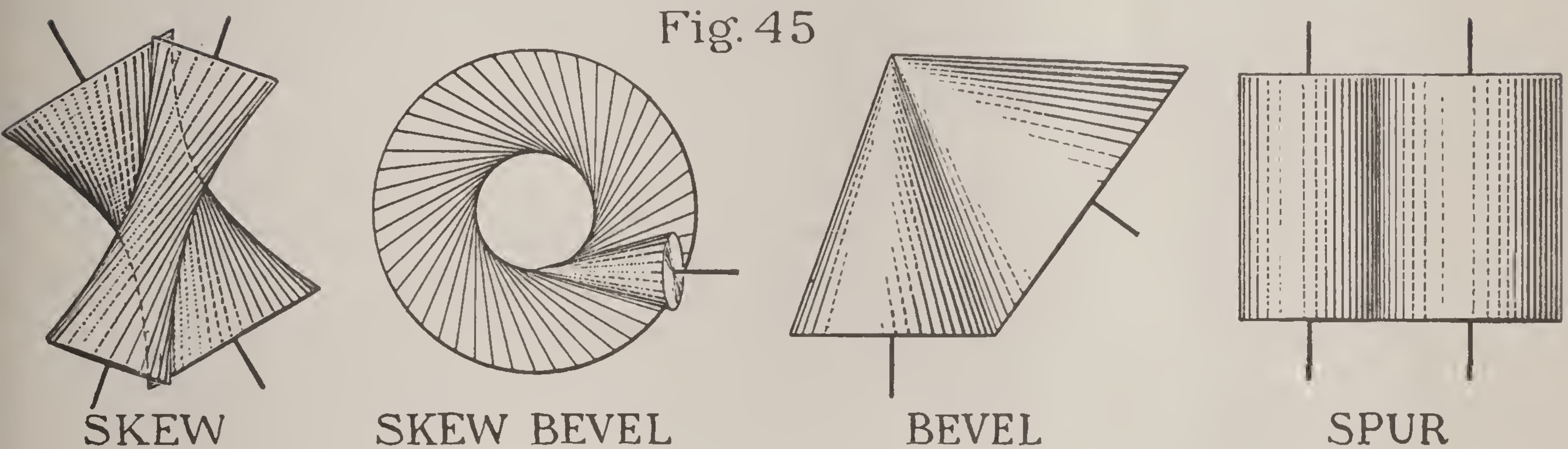
A particular case of the cone rolling and sliding on a developable warped surface is that of a right circular cone rolling and sliding on a circular disc. In this case the axes are neither parallel nor in the same plane ; but the velocity ratio must equal the sine of the angle between the axes, and the line of contact is in the supplement to the same angle. This form is best called the skew bevel gear, and is the exception referred to in the last paragraph.

Two right circular cones rolling on each other are the pitch surfaces of a pair of bevel gears, the axes of which will intersect.

Two right circular cylinders rolling, or rolling and sliding on each other, are the pitch surfaces of a pair of spur gears, the axes of which will be parallel.

Observe that with a given pair of cylinders, the motion can be rolling or sliding or both, which is true of no other form.

Fig. 45



104. Fig. 45 shows the pitch surfaces of the four principal forms of line contact gears.

FORMS OF GEAR TEETH.

105. In the foregoing discussion the gear teeth were supposed to be of infinitesimal dimensions. It would be desirable to know what are the limitations as to form when the dimensions are finite ; but the general case involves some very complicated relations, and in a mathematical treatment there would be no fewer than six independent variables. The problem has never been solved, and it is doubtful whether a solution is possible.

106. In practice little account is made of general theory ; but advantage is taken of a few particular cases that are simple enough to be easily understood and applied.

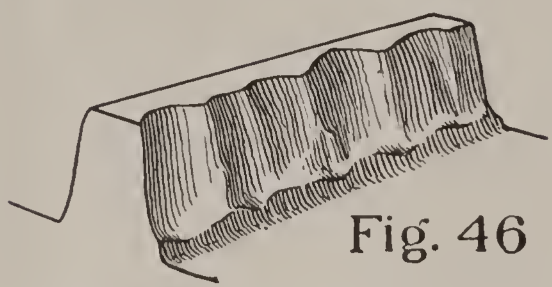


Fig. 46

Fig. 46 will give some idea of how complicated a tooth surface may be, and yet conform to the essential requirements.

107. The more difficult problems in gearing do not admit of graphical treatment ; for instead of plane curves there are undevelopable surfaces to be dealt with. The easiest way to define such a surface as occurs in the perfect bevel gear, for example, is to construct the surface itself. When the surface is formed, as it frequently is by a machine designed especially for the purpose, there is evidently no need of a graphical treatment. The machine is made to cut the gear, and drawing instruments would only be in the way.*

108. Our purpose being to learn the most direct methods of solving graphically such problems as are easiest solved graphically, our field narrows down to include only the spur gear and one or two approximations to other forms.

109. The general condition that must be satisfied by the tooth of a spur gear appears to be as follows :

Let Fig. 47 be a plane normal to the instantaneous axis at the point K. Such a plane will cut the pitch cylinders in two curves, AB and CD, called pitch lines. AB, CD, and EF, are three smooth curves which roll on each other in such manner that their common tangent point is always at K. Let the radii of curvature of the three curves at the point K be R, r, and ρ respectively, the direction in which the radius is measured being indicated by prefixing the sign + or —. Then $\frac{1}{R} > \frac{1}{r} > \frac{1}{\rho}$, and if this expression remains unchanged throughout the rolling, than

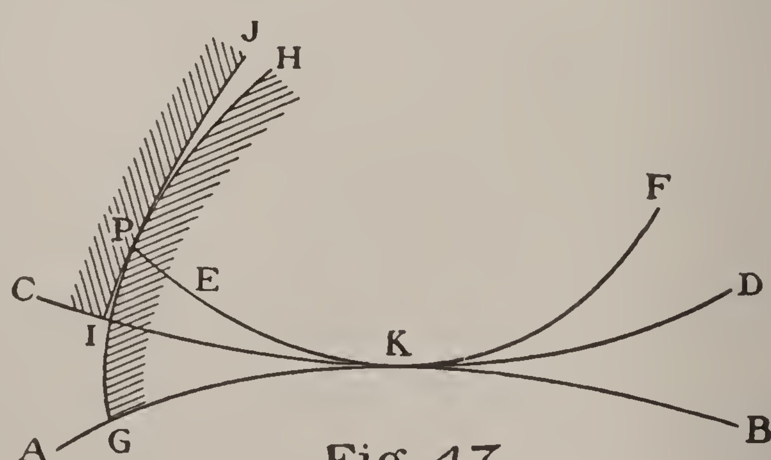


Fig. 47

* For information regarding the production of gears by machinery, see "*Teeth of Gears*," by George B. Grant.

any roulette GH generated by a point P of the curve EF rolling on the curve AB will be a possible tooth profile for the gear of which AB is the pitch line; and the profile of the engaging tooth will be the roulette IJ, generated by the point P, as the curve EF rolls on the pitch line CD of the engaging gear.

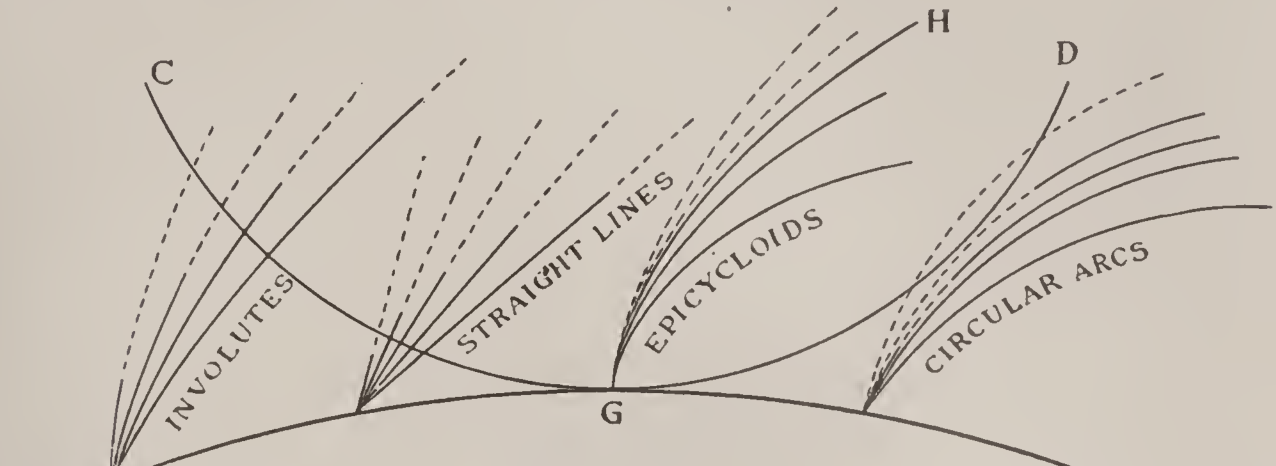


Fig. 48

110. Fig. 48 will give a better idea of the import of the last paragraph. The full lines are possible tooth profiles for the pitch line AB, when rolling on the pitch line CD, while the dotted lines are impossible profiles. The largest possible tooth will have the profile GH, which is an epicycloid generated by rolling CD on AB.

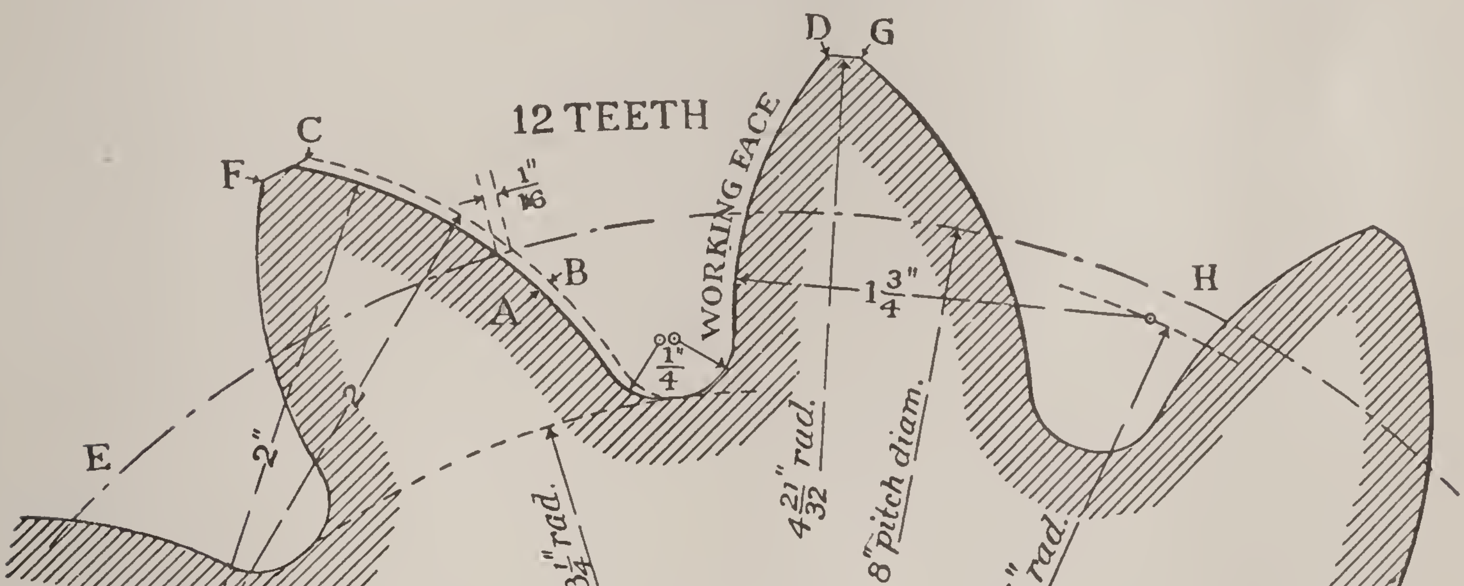


Fig. 49

111. Prob. 7. Fig. 49 shows the tooth profile of a twelve-tooth pinion. The form is arbitrary, but it satisfies the conditions of § 109. It is desired to design a gear with 24 teeth that will properly mesh with this pinion. Make the clearance $\frac{3}{32}''$, and the backlash $\frac{1}{16}''$

112. Draw Fig. 49 by the dimensions given, leaving a space of 8″ above the tops of the teeth. To this figure is to be added the profiles of three teeth of the required gear. The pitch circles are to be placed tangent to each other, and the working faces of the teeth are to be placed in contact, so that the completed figure will represent the teeth in mesh.

113. The back-lash is to be $\frac{1}{16}''$ measured on the pitch line. Let the curve *A*, Fig. 49, be revolved about the center of the pinion through an angle measured by $\frac{1}{16}''$ on the pitch line. It will fall in the position *B*. Draw *B*. This is a temporary alteration, and is to be made on only one of the teeth, as in Fig. 49.

114. The envelope of the curve *CBD* will be the profile of a tooth of the required gear. It is found as follows:

On a piece of sheet celluloid draw an arc of the pitch circle of the required gear (8″ radius). Lay the celluloid over Fig. 49 (on the drawing board), placing the pitch circles tangent to each other at about *E*. Insert a needle at the point of tangency and trace the curve *CBD*, or, more correctly, the curve *FCBDG*, for the points *C* and *D* must be distinct.

Rotate the celluloid about the needle as a pivot until the pitch lines appear to coincide for about $\frac{1}{4}''$ to the right of the needle, insert a second needle at the end of this coincidence ($\frac{1}{4}''$ from the first), and, removing the first, turn the celluloid until the pitch lines appear to be tangent at the second needle point. Again trace the curve *FCBDG*, and so continue alternately rotating and tracing until the pitch lines are tangent near *H*. The envelope of the curves on the celluloid will be the envelope required.

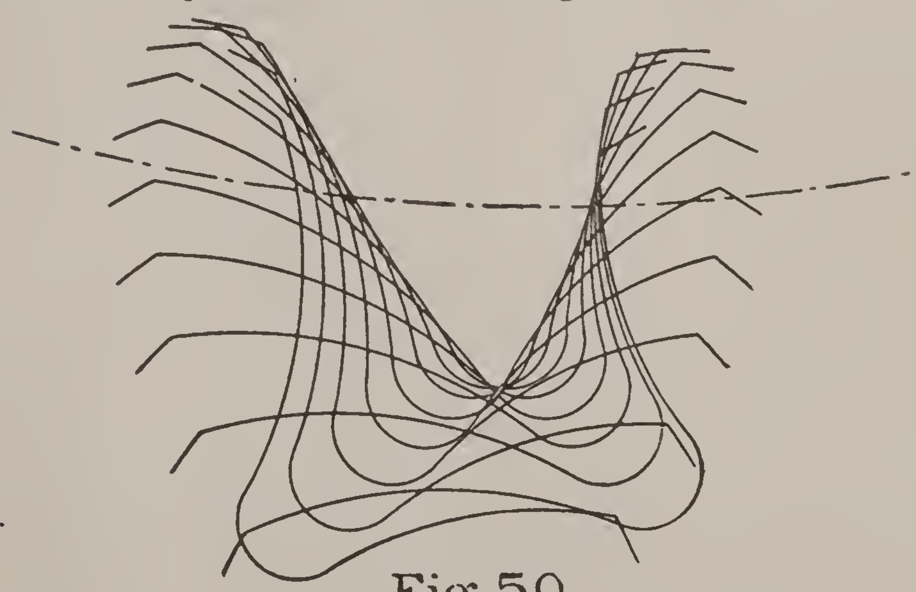


Fig. 50

115. Fig. 50 shows the successive positions of the pinion profile and their relation to the envelope.

Observe that only a small part of each pinion profile is of use in determining the envelope. A little practice will enable one to save time by drawing only the necessary parts.

Lay the celluloid over Fig. 49, placing it in any one of the positions it had during the process of getting the envelope, and transfer the envelope to the drawing board. The transferred curve will be a part of the profile of the required gear in proper mesh with the given pinion profile.

This method of drawing gear teeth is known as the graphical moulding method. As described above, the only theoretical errors are due to the use of a finite number of instantaneous centers, and these errors are far within the errors of workmanship of the most accurate draughtsman or patternmaker. Use transparent sheet celluloid ″.004 thick, with a dull finish on one side. It is much better than tracing paper.

116. The gear teeth will be spaced 15° apart, there being 24 of them. The spacing can be done with the 60° and 45° triangles, as in § 57.

Fig. 51 shows the two profiles in mesh. The envelope found by following the above directions will be the curve *HIJKL*. The

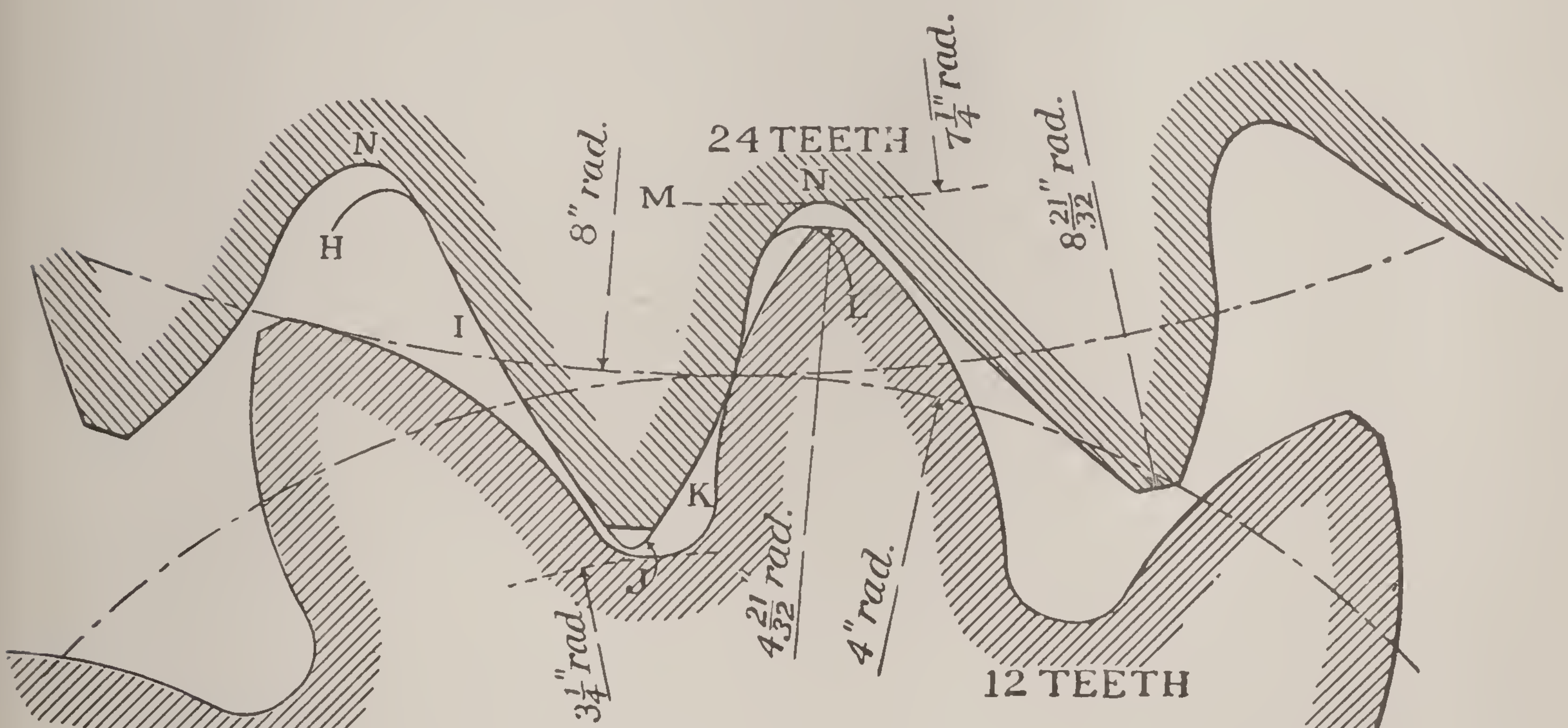


Fig. 51

rounded point *J* is cut off by a circular arc $8\frac{21}{32}$″ radius, and the parts *H* and *L* are replaced by an arbitrary curve *N*, such as will make the root of the tooth touch the circle *M*, $7\frac{1}{4}$″ radius. The clearance is thus made $\frac{3}{32}$″.

117. Besides Fig. 51, which should always be drawn full size, a patternmaker requires a sketch or drawing similar to Fig. 52,

which may conveniently be drawn to a smaller scale. The side view is really necessary only for wheels that have arms; but the section cannot be omitted.

118. Prob. 8. When one of the tooth profiles is a complete circle, the combination is known as a pin gear.

The pin gear can be made to assume a variety of forms by changing the relative positions of the pins and the pitch circles.

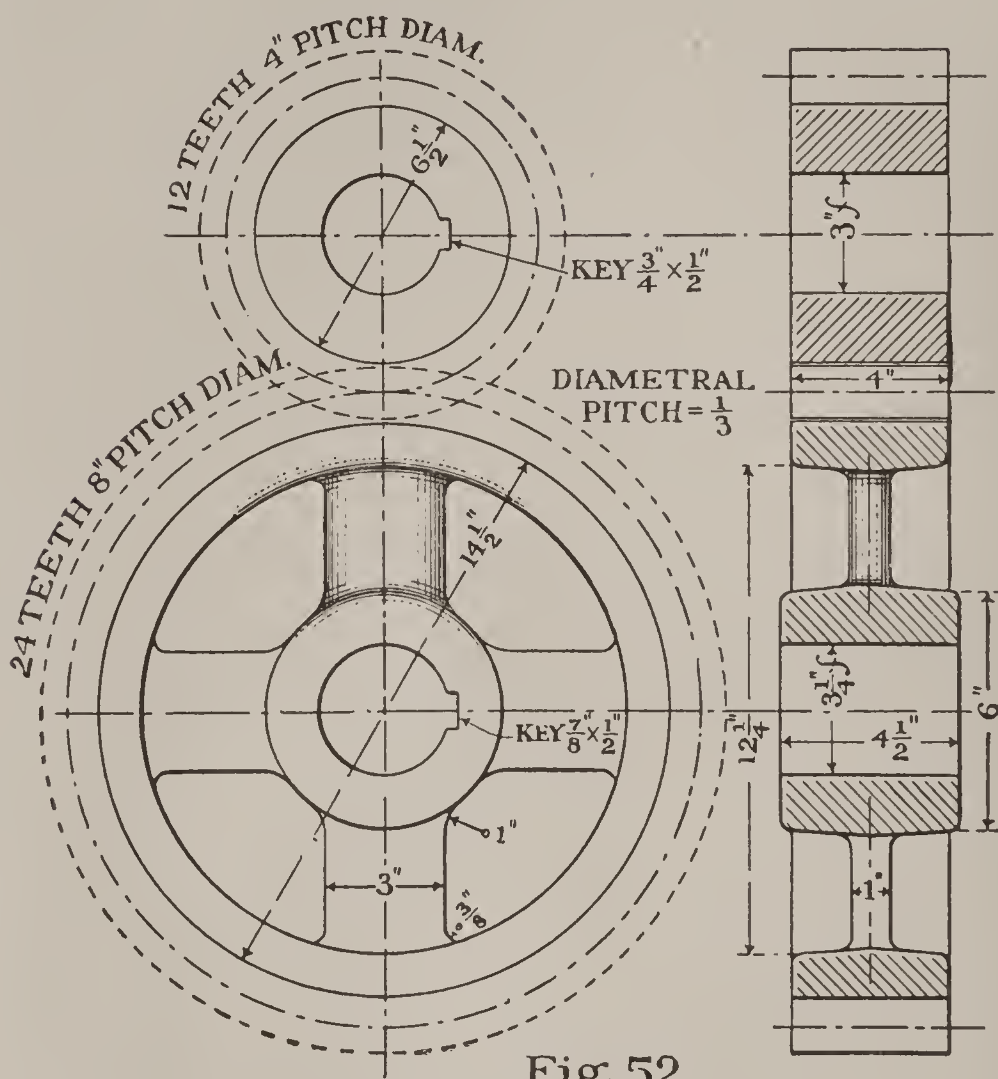


Fig. 52

In Fig. 53, which represents the most common form, the contact occurs on only a small part of the circumference of each pin near the pitch line, and the conditions for correct tooth action (§ 109), require that the center of the pin be placed inside of the pitch line *A*.

119. The distance x, Fig. 54, at which the center must be placed within the pitch line *A*, is found as follows: *CD* is an epicycloid generated by rolling *B* on *A*, and *CE* is the evolute of *CD*. Then, of a pin *F*, whose circumference passes through *C*, the center must

be on or within the curve CE, but not outside of the curve CE. It is possible to find an expression for the minimum value of x in terms of the diameters of the pin and the two pitch circles; but this has never been done.

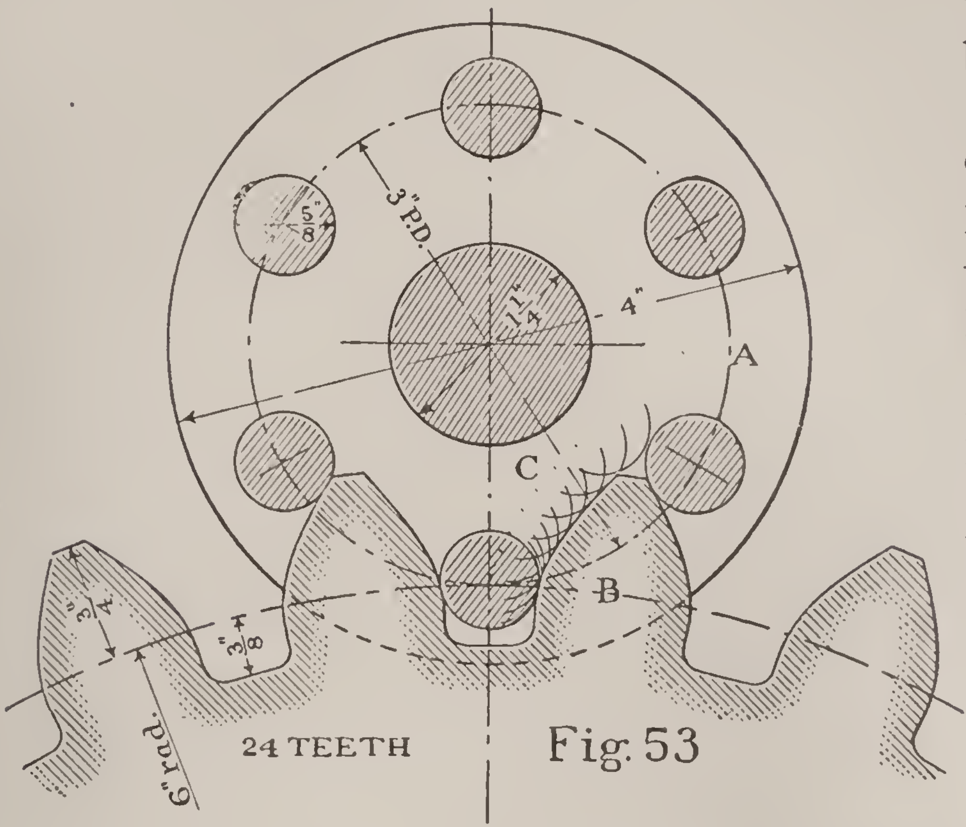


Fig. 53

In Fig. 53 the centers of the pins are placed $\frac{1}{32}''$ inside of the pitch line A.

120. The profile of the tooth that engages with the pin is parallel to the path of the center of the pin, and is found as follows:

Trace on celluloid an arc of the pitch line A, Fig. 53, and prick the position of an adjacent pin center. Roll the traced arc on the pitch line B, taking the instantaneous centers not more than $\frac{1}{8}''$ apart, and prick the successive positions of the pin center. This will give a series of points C in the path of the pin center. With a radius equal to the radius of the pins, draw circular arcs about the points C as centers, and the envelope of these arcs will be the tooth profile required.

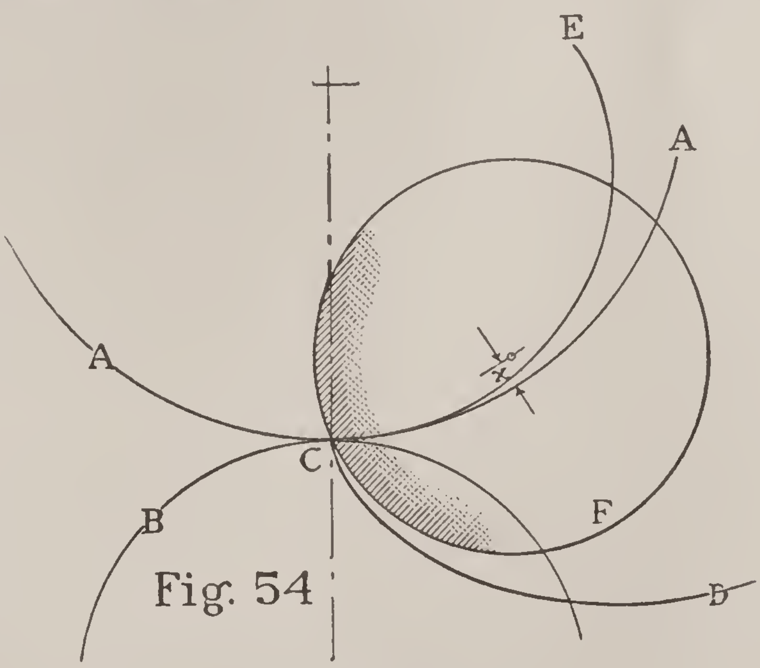


Fig. 54

121. The teeth found as above are made of such length outside of the pitch line that each one continues in contact until after the succeeding tooth begins contact, and the profiles within the pitch line B are made of any simple arbitrary form that will give a sufficient clearance.

122. It may be observed that clock gear teeth are frequently given a form that is theoretically incorrect, for example, like that shown in Fig. 55, and sometimes the profile consists merely of a semicircular end tangent to two straight lines at the sides. We are thus led to the question, When are correct forms necessary?

123. Correctly formed teeth are of the greatest importance in gears that run at high speeds; for at high speeds moving masses will, by virtue of their inertia, exert a considerable force in their endeavor to preserve a constant velocity ratio, and unless the tooth profiles have a corresponding shape, their contacts will consist of a series of blows instead of a steady pressure, the results being noise and rapid wear. For the same reason, the teeth must be equally spaced. The equal spacing is of even greater importance than the correct profiles.

For heavily-loaded gears correct forms of teeth are desirable; since their use will enable the pressure to be divided between two or three teeth, thus lessening the chances of breaking any one tooth.

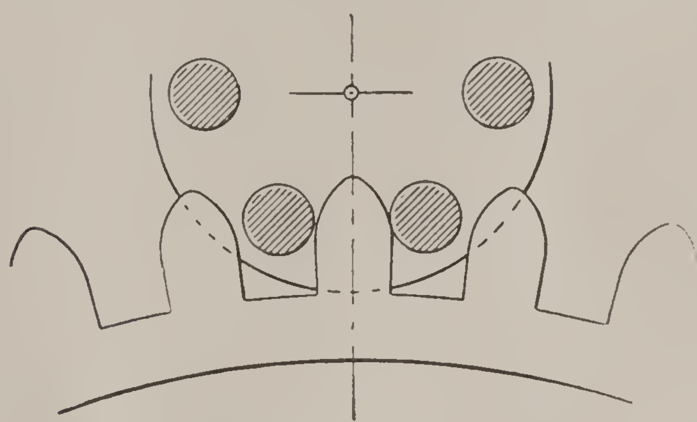


Fig. 55

The function of clock gears is to transmit a moderate pressure from the mainspring to the escapement, and a motion from the escapement to the hands on the dial. In the transmission of the pressure a constant velocity ratio is of no significance. In transmitting the motion, the small variations in the velocity ratio due to the incorrect forms of the teeth are unobjectionable, for the mean velocity ratio is determined absolutely by the numbers of teeth. Next in importance is the equal spacing of the teeth, and least important of all is the correct form of the individual tooth. The clock is merely an instance in which the last condition need not be insisted upon, though there is no doubt but that a clock will work better and last longer by having correctly-formed gear teeth.

124. A chamber wheel, Fig. 56, is a pair of gears inclosed in a fluid-tight case for the purpose of transforming a continuous rotary motion into a fluid displacement, or the reverse. They are used for pumps, blowers, engines, ventilators, water-meters, etc. A great

many forms of the gear or rotating part of the chamber wheel have been devised; but the principle of their action is essentially the same. The gears, by keeping in contact with each other, prevent the fluid from passing between them; while each gear, by revolving in contact with the outer casing, carries the fluid between its projecting teeth from the suction to the delivery side. The chamber gears are usually of such form that they are incapable of driving each other, and so two ordinary spur gears are keyed to the shafts on the outside of the casing.

125. "The theoretical volume of delivery for all forms of chamber wheels, whether continuous or intermittent in delivery, is practically equal to the volume described by the cross-section of a tooth of one of the two gears for each revolution."*

The great weakness of these wheels, which can never be quite overcome, is leakage; and the superiority of any one form will be due to the facility with which machine tools can be made to cut with accuracy its contact surfaces.

126. Prob. 9. Fig. 56 represents the form of chamber wheel known as the Root Blower.

The two chamber gears, A, B, are cast from the same pattern. Their outer curved surfaces are finished; also their ends, which are perfectly flat, and the short hubs projecting inwards from the ends are bored to $1\frac{1}{8}''$ diam. for the shafts. The two semicylindrical shells C, D are cast from the same pattern, and finished by first planing the flat surfaces a, b, c, d. The two castings are then bolted together (a in contact with b, and c in contact with d), and the insides of both are finished at one boring, and the flanges are faced. The two end plates, E, F, are cast from the same pattern; their flat surfaces, which come in contact with the ends of the chamber gears, are planed, and the holes are bored for the adjustable bushings, G, G, G, G. The end plates and the semicylindrical shells are fastened together by 24 bolts $\frac{3}{8}''$ diam. Of the 24 bolts, at least eight must be "finished" or "fitted," so that when the parts are fastened together the inner surfaces of the shells shall be exactly concentric with the bearings in the end plates. The remaining 16 bolts may be left "rough" or "black." The upper shaft is lengthened at H for a driving pulley; otherwise the two shafts are alike. The two gears, I, J, have cut teeth, and the blanks are cast from the same pattern. All parts are of cast iron, except the shafts and

*Reuleaux: *The Constructor* (Suplee's trans.), p. 220.

bolts, which are of wrought iron, and the bushings, which are of brass.

Besides the two views shown in Fig. 56, draw an end elevation at K, looking in the direction of the arrow, and leaving out one of

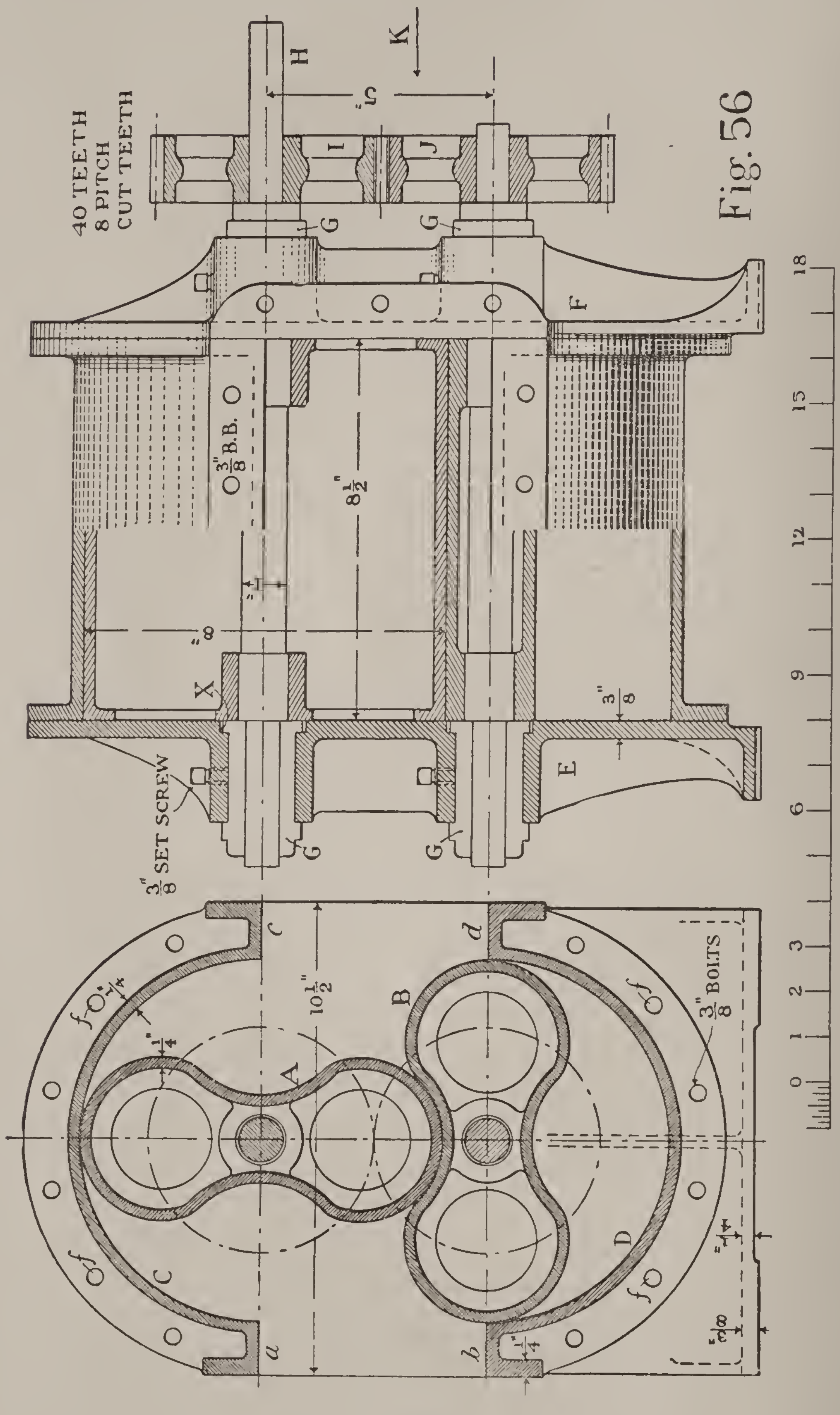


Fig. 56

the gear wheels, I, J. Also a full size cross-section of a chamber gear, as in Fig. 57, and detail views of the adjustable bushing, as in Fig. 58.

127. In Fig. 57 the curves are all circular arcs, except that portion of the profile which is within the pitch circle.

Draw Fig. 50, excepting the irregular curves, by the dimensions given, and then find the irregular profiles, as follows:

Trace on a piece of celluloid the center line AB, and the pitch line L, and prick the center N. Place the celluloid so that the tracing of AB coincides with CD, and the tracing of the pitch line is externally tangent to L. The pricked center will fall at n. From this position we are to roll the pitch line of the celluloid on the pitch line of the gear, and determine points in the path of n. The describing point n is on the celluloid. Insert a needle at p, and around p as a center rotate the celluloid through a small angle. Insert a second needle at the point where the pitch circles intersect. Remove the first needle, and turn the celluloid about the second needle until the pitch circles are tangent at the second needle. Prick the position of n from the celluloid. By repeating the process any number of points can be found in the path of n. Observe, that in this case the pitch circles are of equal curvature, and hence the above method is theoretically correct, no matter how great the angle through which the celluloid is turned at each instantaneous center.

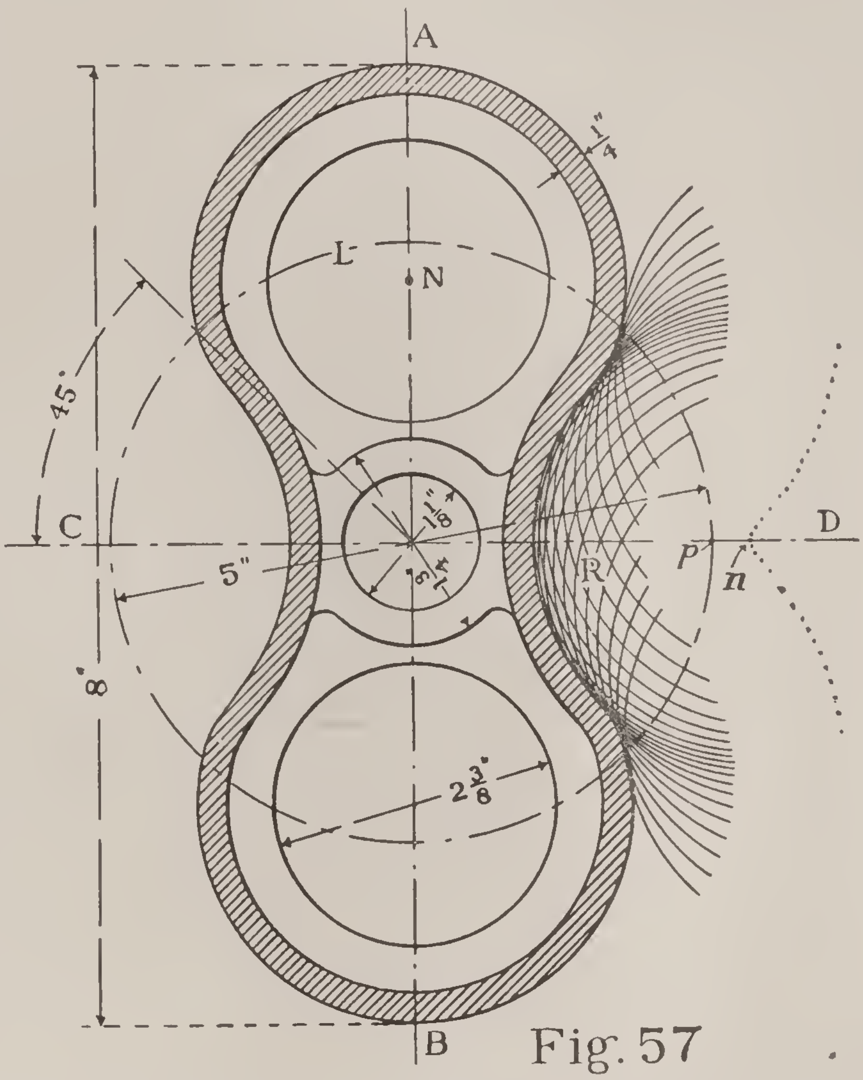


Fig. 57

The path of n will be a hypotrochoid, and the required profile is parallel to this path. Take the radius NA in the compasses, and,

with centers at the various positions of n, draw the circular arcs R. The envelope of these arcs is the required profile. It will be seen that this chamber gear is very similar to the pin gear of Prob. 8, the profile being parallel to the path of a point in both cases.

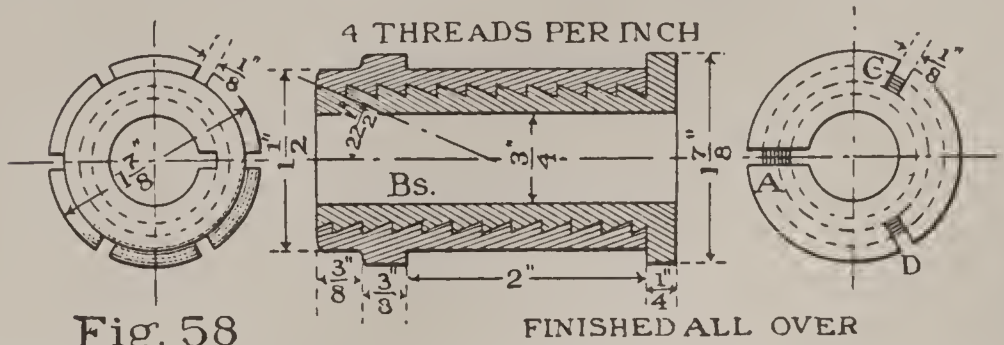


Fig. 58

128. To take up the wear in a bearing without disturbing the position of the center of the shaft, a special adjustable bushing, Fig. 58, is used. It consists of two sleeves screwed together by a buttress thread. The outside diameter of the thread is $1\frac{1}{4}''$. The inner sleeve is slotted the whole length at A, and its flange is slotted at C and D, to make it elastic. The outer sleeve has six notches by which it can be turned with a spanner wrench. A dowel on the inside of the casing (X, Fig. 56) engages one of the slots A, C, or D, Fig. 58, and prevents the bushing from turning with the shaft, at the same time allowing the bushing to be placed in any one of three positions to distribute the wear. A set screw, Fig. 56, holds the outer sleeve after it is tightened.

Devise some way of continuously lubricating this bearing.